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Aerial Navigation

By Lillian Moore

Far Rockaway High School, New York, New York

At the present time mathematics for defense is the primary aim of the alert teacher of mathematics. Teachers must engage enthusiastically in the national civilian defense training program. The secondary-school teacher of the social sciences is interested principally in democratic principles, the development of loyalty and cooperation as citizens of a representative democracy. The teacher of the physical sciences is interested in giving students technical knowledge which will enable them to work intelligently in airplane factories, hospitals, shipyards, at the drafting board, on the assembly line, typing, filing, or to prepare for active duty in the army or navy. The teacher of mathematics should realize the importance of mathematical training in the defense program.

The high school boy or girl is vitally interested in national defense. First aid courses under the sponsorship of the Red Cross Association are extremely popular. As general assistant of an evening high school in Brooklyn, I am besieged by young people now employed during the day who desire to prepare for nursing, coast guard duty, engineering or aeronautical work. They register for the courses in mathematics, chemistry or physics which they failed to take in day high school. They realize that a knowledge of mathematics especially is an indispensable prerequisite for such work. Educational and vocational guidance is greatly needed and frequently neglected. How can the high school teacher remedy this condition?

He should be familiar with the defense series publications of the United States Office of Education. He should have a knowledge of the experimental work in applied mechanics and the application of higher mathematics to defense problems being accomplished at Brown University. He should be familiar with the report of the Subcommittee on Education for Service of the War Preparedness Committee of the American Mathematical Society and the Mathematical Association of America on *Mathematical Education for Defense*. Its recommendations should be followed, especially that of advertising the "utility of mathematics in industry, government, the professions, and military science." One suggestion of the committee, to stress the importance of applications of secondary mathematics, can be followed with highly satisfactory results by referring to the mathematics of aviation.

One requisite for national defense is the creation of a citizenry cognizant of the importance of aircraft in modern warfare. A knowledge of aerial navigation is the best possible preparation for making American youth air-minded. Being concerned about national defense, I decided to introduce a course in aerial navigation as an extra-curricular activity in the New

York City high school where I teach mathematics during the day. Incidentally I hold a Civil Aeronautics Administration ground instructor's certificate. The course was instituted in September, 1941, and has proved successful beyond all expectations. Registration was limited to those students who had completed courses in elementary algebra, plane geometry and intermediate algebra. Any additional mathematical knowledge required was developed during the course as the need therefor arose during the lectures.

The course has been fun for both pupils and instructor. It has widened the horizon of their interests and knowledge. It has resulted in a more intelligent appraisal of newspaper and magazine articles on aircraft accidents, aircraft development and air progress during the present war. One student remarked last week when showing the class a scrapbook on the development of aircraft design throughout the thirty-eight years of its history that he had predicted the collapse of the Italian air forces in the Mediterranean as a result of a study of his scrapbook illustrations. Another young man left recently for the Pensacola Training School. A third pupil will combine his remarkable talent for photography and his interest in flying as preparation for a career in aerial photography. Because the course has proved to be so valuable, I am including an outline of the lectures given, with the hope that it will prove suggestive to other teachers of mathematics throughout the country. As a practical application of arithmetic, algebra, geometry and trigonometry, the work in aerial navigation is ideal. The possibilities of such a course are unlimited. Symbolically, if not literally, the pupils are developing wings for victory.

Course in Aerial Navigation

I. Introduction
  1. Definition of aerial navigation
  2. Problems of air navigation
    a. Theory of flight
      (1) Historical development
      (2) Glider, balloon, power-driven airplane
    b. Aircraft construction
      (1) Airplane design — fuselage, wings, control surfaces, landing gear
      (2) Engine and propellers
      (3) Forces acting on plane—lift, gravity, thrust, drag
    c. Aids to navigation
    d. Traffic control
    e. Use of charts
      (1) Map projections — Mercator, Lambert conformal conic, gnomonic
      (2) Advantages and disadvantages of various types
      (3) Latitude and longitude
  3. Types of air navigation
    a. Contact flying
    b. Dead reckoning
    c. Radio navigation
    d. Celestial navigation
  4. Dependence of one method upon others

II. Instruments
  1. Those essential for navigation
    a. Clock
    b. Altimeter
      (1) Description
      (2) Altimeter errors
    c. Compass
      (1) Types—magnetic-needle, earth-inductor
      (2) Compass errors
        (a) Variation
        (b) Deviation
        (c) Instrumental errors
        (d) Northerly and southerly turning errors
        (e) Flight errors
      (3) Compensating for errors
    d. Airspeed indicator
      (1) Description
      (2) Calibration—correction necessary
    e. Drift sight
      (1) Description
      (2) Use
  2. Other instruments
    a. Directional gyro
    b. Flight indicator or artificial horizon
    c. Bank-and-turn indicator
    d. Rate-of-climb indicator
    e. Radio instruments
      (1) Radio receiver
      (2) Radio compass
      (3) Radio direction finder
      (4) Radio altimeter
    f. Automatic pilot
    g. Engine instruments

III. Meteorological knowledge essential for safe flying
  1. Structure of air

2. Stratification of air
   a. Troposphere
   b. Stratosphere
   c. Tropopause
3. Weather conditions of interest to aircraft pilot
   a. Temperature
   b. Pressure
   c. Wind
   d. Humidity
   e. Precipitation
   f. Clouds
   g. Visibility
4. Wind systems
   a. Forces moving air
      (1) Pressure
      (2) Friction
      (3) Earth's rotation
      (4) Gravity
   b. Pressure areas
      (1) Highs
      (2) Lows
   c. Pressure distribution
      (1) Prevailing winds
      (2) Monsoons
   d. Turbulence
      (1) Squalls
      (2) Eddies
5. Air masses
   a. Types
   b. Fronts
   c. Properties
6. Weather blackout because of war
7. Weather forecasting

IV. Contact flying
1. Procedure
2. Maps and charts
   a. Topographic information
   b. Aeronautical data
3. Flying a range, keeping two objects in line
4. Line of position and fix
5. Determination of ground speed

V. Dead reckoning
1. Advantages and disadvantages
2. Basic problems
   a. Case I: When planning flight, to determine from chart distance and compass heading to be followed
   b. Case II: While in flight, to determine track and position of plane from observed compass heading and airspeed
3. Special problems
   a. Returning to the intended track
   b. Radius of action
      (1) Returning to same base
      (2) Returning to different base
   c. Interception
   d. Determination of wind drift while in flight

VI. Radio navigation
1. Importance as supplement to dead reckoning
2. Basic principles of radio transmission
   a. Radio signals
   b. Errors in radio direction finding
3. Radio range system
   a. Flying a beam
   b. Effect of wind
   c. Methods of orientation
      (1) Ninety degree method
      (2) Fade-out method
      (3) Parallel method
      (4) Combination method
   d. Marker beacons
      (1) Types—M type, fan marker, zone marker
      (2) Cone of silence
4. Radio direction finding
   a. Loop aerial
   b. Radio compass bearing
   c. Homing
5. Plotting radio bearings
   a. Relative bearing
   b. True bearing
6. Instrument landings

VII. Celestial navigation
1. Preliminary drill on use of tables and sextant
2. The astronomical triangle
3. Time—apparent solar, mean solar, standard, sidereal
4. Plotting course on Mercator projection chart
5. Problems
   a. To determine great-circle distance between two points
   b. To find vertex of great circle
   c. To find true course and distance between two points
   d. To find hour angles of stars and planets
   e. To determine latitude, using observation on Polaris
   f. To determine latitude from transit of star
   g. To find longitude from sun
   h. To determine line of position, using star observation
      (1) Azimuth
      (2) Altitude intercept
   i. To determine position from astronomic sights of stars or moon.

Mathematics and Science in Aeronautics

By Arthur S. Otis
Technical Member of the Institute of the Aeronautical Sciences

The teaching of mathematics and the teaching of science are becoming yearly more closely bound together for a number of reasons. One of these is, no doubt, the fact that science cannot be understood without an understanding of mathematics, and it is often easier and more direct to teach the mathematics of science along with the science than to do so separately in uncorrelated courses. A second reason is that the mathematics itself is much more easily and thoroughly understood by students if applied, and since science offers one of the best possible applications of mathematics, it becomes necessary to discuss a great many of the scientific applications of mathematics in the mathematics class. A third reason is that we are unable to teach all branches of mathematics under any circumstances, and it would seem more practical, therefore, to teach that particular mathematics which we believe most likely to be used. And if the mathematics of science is taught with the science, we know at least that the mathematics that is taught in such a course will be used if the science is used.

Articles on the mathematics of aviation have appeared from time to time in The Mathematics Teacher. In view of the growing coordination of mathematics and science, it seems desirable to acquaint mathematics teachers with the applications of both mathematics and science in aeronautics, particularly in view of the prominence of aeronautics in our preparedness program, and indeed in the future of our transportation.

At the expense of some slight duplication of what has appeared in other articles, the following outline is given of the laws and principles of science and mathematics that are applied in aeronautics.

The outline is by no means exhaustive. It is confined to topics suitable for discussion in a high school course in aeronautics, or in high school courses in mathematics and science, and it presumes to cover only a portion of that field. For example, the whole field of airplane design, construction, maintenance and repair, and engine design, construction, maintenance and repair are untouched in this outline; also are the whole field of the physiological aspects of flying and celestial navigation. Moreover, the applications mentioned in this outline may be thought of as referring merely to private flying—indeed peacetime private flying. With very few exceptions, those aspects of aeronautics peculiar to commercial or military flying have been disregarded. It goes without saying, however, that all the topics mentioned in the outline are necessary or at least important to be understood by a flyer training for national military preparedness.

This outline is given, not to be read carefully or studied, but merely to give a general impression of the wide use of science and mathematics in aeronautics, even in a limited area of peacetime private flying, suitable for study by high school students.

Outline of Applications of Mathematics and Science in a Limited Field of Aeronautics

A. Principles involved in aviation instruments
   1. Centrifugal force
   2. Static and dynamic air pressure
   3. Decrease of barometric pressure with altitude
   4. Magnetism

B. Airfoils
   1. Symmetry
   2. Measurement of angles—of incidence—of attack—of relative wind
   3. Bernoulli's law
      a. Kinetic, potential, and pressure energy
      b. Conversion of energy
      c. Streamline flow
      d. Formula for Bernoulli's law
      e. The Venturi tube

4. Newton's laws of motion—involving inertia, momentum, velocity, force, etc.
5. Induced and dynamic lift
6. Summation of forces into a resultant force

C. Forces acting on a plane
1. Equilibrium of forces
2. Representation of forces by vectors
3. Acceleration caused by unbalanced forces
4. Weight—thrust—lift—drag
5. Parallelogram law of forces
6. A plane in straight level flight, climbing, gliding
7. Change of forces on a plane taking off

D. Stability in a plane
1. Equilibrium—neutral—stable—unstable
2. Restoring tendency
3. Damping tendency—of fin, rudder, sweepback, dihedral, etc.

E. Lift and drag
1. Coefficients of lift and drag
2. Variation of lift and drag
   a. As area of wing
   b. As density
   c. As square of speed
   d. With angle of attack
   e. Reading a graph showing this variation
3. Slug, the unit of mass
4. Density—variation with altitude
5. Parasite drag—formula for
6. Relation between miles per hour and feet per second

F. Speed and power
1. Formula for required velocity
   a. Finding square root by approximation
   b. Finding square root by a table
2. Relation of thrust to weight
3. Lift-over-drag ratio
4. Power—foot pounds—horsepower
   a. Formula for
   b. Variation with angle of attack

G. Airplane performance
1. Change of required velocity and horsepower
   a. With change of density
   b. With change of altitude—square root of a decimal
   c. With change of wing area
   d. With change of weight
2. Wing loading (weight per unit of wing area)

H. Engines and propellers
1. The 4-cycle engine
2. Torque
3. Variation of available HP with RPM
4. Propeller blade angle—pitch
5. The helical of motion of a propeller tip

I. Climbing and circling
1. Foot pounds per second in climbing
2. Change of rate of climbing with altitude
3. Centrifugal force in circling—parallelogram law of forces—theorem of Pythagoras
4. Pivotal altitude—formula for

J. Trigonometry of aeronautics
1. Finding an angle of bank
2. Finding the height of a cloud bank
3. Finding the component of gravity in a glide
4. Finding the height of a balloon
5. Finding the pitch of a propeller
6. Finding one's distance from a landmark by angle of depression
7. Forward motion of a trajectory
8. Etc., etc.

K. Navigational charts
1. Contours
2. Conformal conic projection

L. Dead reckoning
1. Computing distances by a chart
2. Measuring directions
3. Magnetic north — variation — isogonic lines
4. Finding magnetic course—compass course — use of deviation charts

M. Wind drift
1. Use of vectors to represent motion
2. Air speed versus ground speed—wind drift angle—adding vectors
3. Finding a heading—a track—a wind direction
4. Finding the ground speed—the wind velocity—measuring drift
5. Drawing to scale

N. Interception
1. Nautical miles—knots
2. The principle of the constant bearing
3. Representing time on a graph
4. Measuring time by distance

O. Finding a radius of action
1. Concept of locus—relative wind angle
2. Drawing a perpendicular bisector
3. Drawing an ellipse
4. Formula for radius of action

P. Flying the figure eight
1. Drawing a graph
2. Concept of relativity—air path versus ground path

Q. Downdrafts—adding vectors of motion

R. Radio ranges
1. Direction finders
2. Establishing a fix

S. The earth and its atmosphere
1. Troposphere and stratosphere
2. Air masses and their sources
3. Weight of air
4. Adiabatic lapse rates
   a. Barometer
   b. Millibars

c. Boyle's law
d. Fahrenheit and Centigrade thermometers
e. Charles' law
f. Gay-Lussac's law
g. Absolute temperature
h. Expansion of air with lifting
i. Cooling of lifted and expanded air
j. General atmospheric lapse rate
k. Stable and unstable air
l. Relativity in warmth

T. Cloud formation and precipitation
1. Vapor capacity
a. Grams and kilograms
b. Variation with temperature
c. Variation with pressure
d. Saturation
2. Condensation—dew point
3. Latent heat of condensation—effect upon cooling with lifting—pseudo-adiabatic process
4. Adiabatic chart—relative humidity

U. Air-mass sources, characteristics, and motions
1. Terrestrial radiation
2. Clockwise and counter-clockwise deflection of wind

V. Weather forecasting
1. The wave theory
2. The wind relative to a wave—relative to the ground
3. Lows and highs
4. Isobars
5. Pressure gradients
6. Fronts — clouds — fog — icing — turbulence — high winds — thunderstorm — tornado

W. Instrument flying
1. Principle of the climb indicator—capillary leak
2. Use of the gyroscope—resistance to rotation of axis—precession
a. In the rate-of-turn indicator
b. In the directional gyro
c. In the "artificial horizon" instrument
3. Weather reporting

X. The physics of high altitude flying — pressure cabins

It goes without saying, perhaps, that if we were to pursue the subject of aeronautics into its more mechanical aspects and into the fields beyond the range of the high school student, including, for example, the design and construction of aeronautical instruments, streamlining of airfoils, airplane construction with a study of stresses and strains, cost of materials, etc., wind-tunnel experiments on lift and drag, the designing and construction of airplane engines to increase the ratio of power to weight, the design of automatic variable pitch propellers, the changes in design of planes to accomplish high speed and great size, radio communication and the design of electrical instruments for navigation and the guidance of airplanes, the whole science of meteorology in its relation to weather forecasting, the science of instrument flying and the instruments involved, the effect upon the human body of high-speed climbing, diving, etc., to say nothing of the vast experimentation involved in designing unconventional craft such a helicopters, autogyros, etc., we see that the mathematics an science of aeronautics are almost limitless.

If the above outline and accompanying comments have served to demonstrate to the reader, first, how vast the number of applications in mathematics and science in aeronautics, is, and second, what a wide variety of such applications there are even in a most elementary course of aeronautics, the principal purpose of this article will have been achieved. The purpose of the remaining portion of the article will be, therefore, merely to expand a very few of the topics mentioned in the outline in order to give the reader a more magnified view of these topics, and to show in a general way how much mathematics and science may be encompassed by one topic in the outline.

Bernoulli's Law

Let us take a close-up view of the mathematics and science involved in an understanding of Bernoulli's law in its application to the so-called induced lift of an airfoil.

By airfoil is meant an object, such as the wing of an airplane, which has been streamlined; that is, constructed to permit a free and unobstructed flow of air past it in such a manner as to be aerodynamically significant, as we say—that is, in a way which is important with reference to the

flight of the airplane. The principal airfoils of an airplane are the wings, propeller, and the tail surfaces.

Fig. 1 shows a cross-section of the wing of an airplane in flight when the wing is at "zero angle of attack." An airfoil section is said to be at zero angle of attack when its chord (in this case the straight underside) is parallel to the direction of flow of air. (We speak of the direction of flow of air with reference to the airfoil, even though actually in flight it is the airfoil that is moving through the air which may be stationary. But the student learns early in his study of aerodynamics that airflow is a matter of relativity and it makes no difference whether the air is flowing past an airfoil or the airfoil is moving through stationary air; all forces acting relative to the airfoil are identical in the two cases.)

An interesting fact which is surprising to nearly every student of aerodynamics is that an airfoil may have actual lift even when at zero angle of attack. This lift is due to the fact that although the pressure on the lower surface of the airfoil may be little or no greater than normal atmospheric pressure in the vicinity of the airfoil, nevertheless the pressure on the upper surface of the airfoil is reduced, and the difference between these pressures which constitutes excess pressure on the lower surface affords lift. It is the study of Bernoulli's law that explains this reduction in pressure on the upper surface of the airfoil.

Bernoulli's law has to do with the relation between changes in velocity and changes in pressure of a body of particles of air in an airflow. Bernoulli's law is in effect a statement of a law of conservation of energy. It is a mathematical statement of the fact that the sum of the kinetic energy, the pressure energy, and the potential energy of position of a body of air is constant. The application of this law to aerodynamics lies in the corollary to the effect that if the potential energy of a body of air remains constant, any increase in kinetic energy is had at the expense of pressure energy. In simple language Bernoulli's law boils down to this. "In a streamline flow the greater the speed of the air the less the pressure of the air, and the less the speed of the air the greater the pressure."

The application of Bernoulli's law to the airfoil lies in the fact that because of the curved shape of the upper surface of the airfoil shown in Fig. 1, the air has to travel

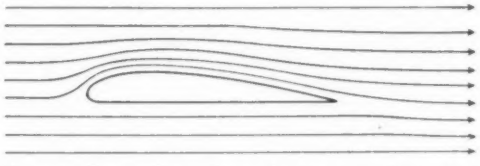

FIG. 1. Streamline flow of air around a wing section at zero angle of attack.

a longer distance to go over the top of the airfoil in the manner shown than other air travels below the airfoil in the same length of time. This means that the air above the airfoil is traveling more rapidly, and since "the greater the speed the less the pressure," we have less pressure of air along most of the upper surface of the airfoil, and consequently obtain lift as explained above.

The student may perform some simple but interesting experiments demonstrating Bernoulli's law. Some of these are shown in Fig. 2, which show (*a*) drawing a small pith ball over to an air stream, (*b*) the inability to blow a small card away from the end of a spool, (*c*) balancing a small pith ball on an air stream, and (*d*) lifting a curved strip of card by means of a stream of air.

FORCES ACTING ON A PLANE

In the study of the forces acting on a plane in flight is to be found a useful and interesting application of the study of forces, their representation by vectors, their combinations and resolutions by means of the parallelogram law of forces, and of Newton's laws of motion of balanced forces, equilibrium, etc.

Fig. 3 shows how all the forces acting on the section of an airfoil can be repre-

sented as one resulting force by a single vector, issuing from what is known as the center of pressure of the airfoil section.

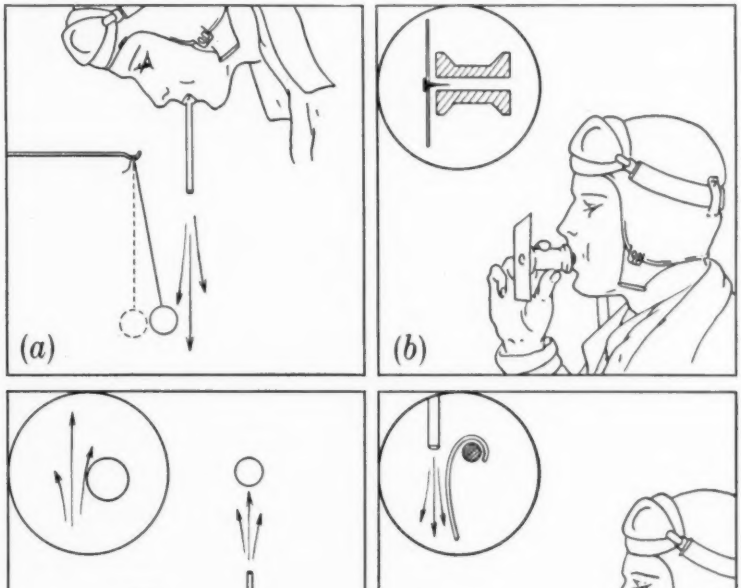

FIG. 2. Experiments illustrating Bernoulli's law

The sum of these resultant forces on the several sections of the airfoil can be assumed to be combined and represented by a single vector, OW, in Fig. 4, issuing from the center of gravity of the airplane. The total force, OW, on the wings of the airplane is known as lift. The vector OP represents the pull of the propeller, technically known as thrust; the vector OG represents the force of gravity acting on the airplane, and the force of the vector OD_p represents the so-called parasite drag of the airplane; that is, the friction of the air on the airplane other than its wings.

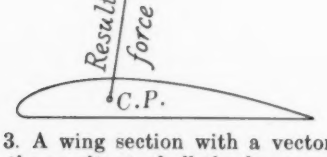

FIG. 3. A wing section with a vector representing the resultant of all the forces acting on the section.

Fig. 5 shows how, by resolving the total lift OW into two component forces, one vertical (OL) known as sustentation, and one horizontal (OD_w) known as wing drag,

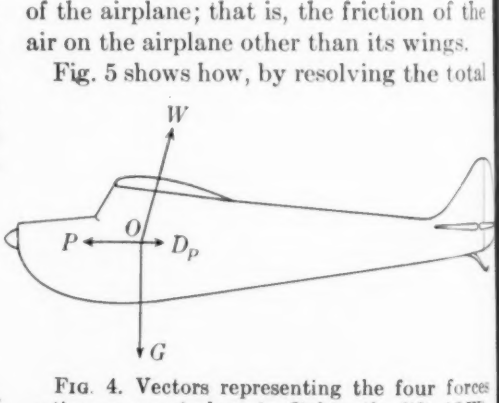

FIG. 4. Vectors representing the four forces acting on an airplane in flight—the lift (OW), gravity (OG), thrust (propeller pull, OP) and "parasite drag" (OD_p).

by substituting these components for OW in the system of forces of Fig. 4, and adding the horizontal forces acting in the same direction, we obtain at (*d*) the completely balanced forces which are necessary to be acting upon a plane in order that it shall move in a straight line at uniform speed.

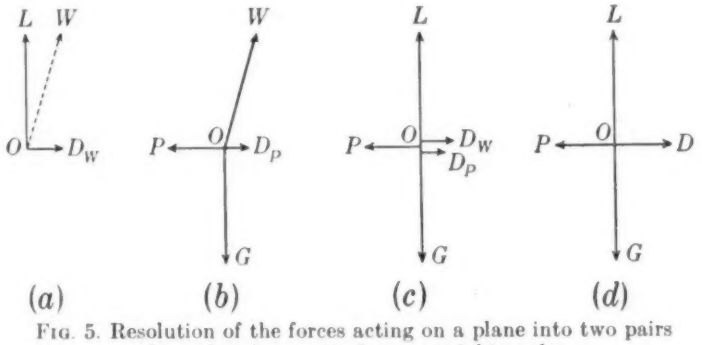

FIG. 5. Resolution of the forces acting on a plane into two pairs of equal and opposite forces at right angles.

Similar studies may be made of course of the forces acting on the plane (1) as it is picking up speed when rolling on the ground, (2) climbing, (3) while gliding, and (4) while circling, etc.

In a circling plane we have of course in addition to the four forces shown at (*d*) in Fig. 5, the additional force, centrifugal force. This is counteracted by "banking" the plane.

It is interesting in this connection to learn why more power is needed while circling and why the angle of attack must be increased in order that the lift will be augmented sufficiently so that it can be divided into two components, a vertical component, sustentation, sufficient to overcome gravity, and a horizontal component sufficient to overcome the centrifugal force of the turn. We can merely touch upon these matters here.

A second interesting aspect of the forces acting on a plane has to do with stability in flight. Fig. 6 shows three main forces acting on an airplane which is in the nature of a lever. *C* represents its center of gravity and the downward pointing vector from *C* represents the weight of the plane. Point *L* is the center of lift of the wings, and the vector from *L* represents the force of sustentation acting upon the airplane. The downwash from the wings strikes the upper surface of the stabilizer *S*, and produces a slight downward force at this point. These forces stabilize the plane in flight in the following manner.

If the plane is accidentally blown into a nose-up position, and therefore begins to climb slightly, its velocity is reduced because part of the horsepower formerly used merely to overcome air friction is now required to lift the plane in the air. This reduction in forward speed in turn reduces the amount of downwash and the amount of downward pressure on the stabilizer.

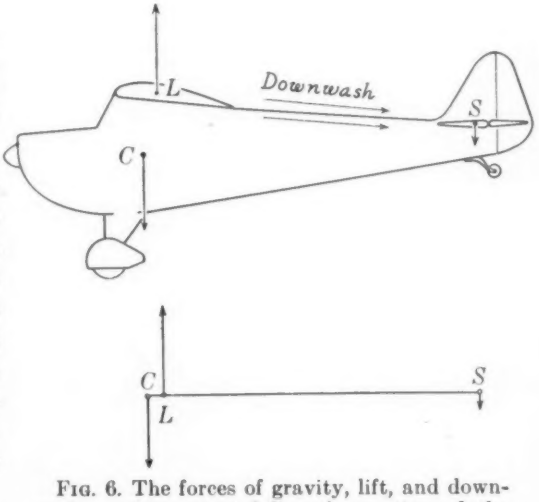

FIG. 6. The forces of gravity, lift, and downwash combine to stabilize the motion of the plane with reference to pitching (up and down motion of the nose).

A lever *CLS* having been in equilibrium in level flight is now thrown out of balance and the remaining "couple" consisting of the vectors issuing from *C* and *L* has the effect of rotating the plane on its "lateral axis" so that it noses down slightly. In other words, it tends to be restored to level flight. Thereupon, its forward speed picks up and the lever *CLS* resumes its position of balance.

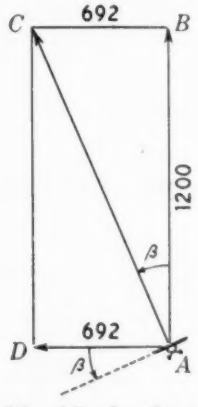

FIG. 7. The lift, *AC*, of a plane flying a curve in a banked altitude, resolved into a force of sustentation, *AB*, and a centripetal force, *AD*.

In a similar manner it can be shown that if the plane accidentally gets pointed downward, the forward speed of the plane increases, and downwash increases, and the force at *S* increases, which tends to lower the tail again and thus restore the plane to level.

These illustrations of course merely touch upon the broad subject of forces acting on a plane.

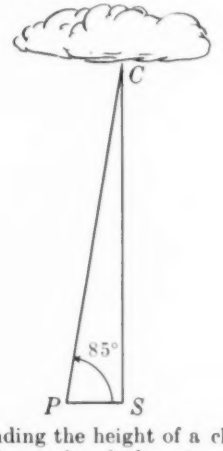

FIG. 8. Finding the height of a cloud base by measuring the angle of elevation of a searchlight spot, *C*, on the cloud base from point *P*, at a known distance from the vertical searchlight beam *SC*.

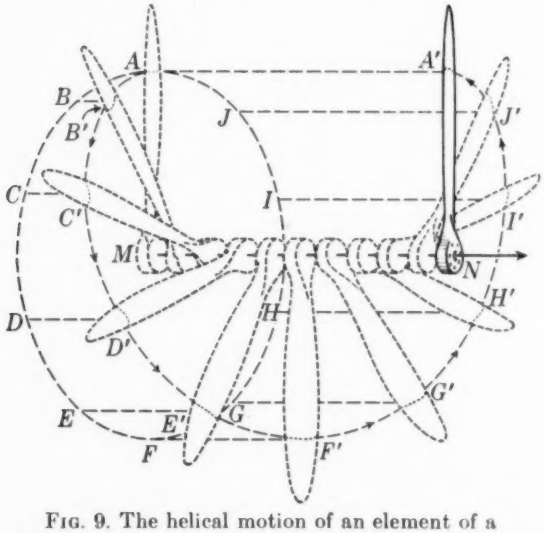

FIG. 9. The helical motion of an element of a propeller blade.

THE SIMPLE TRIGONOMETRY OF AERONAUTICS

There is space for only two or three illustrations of the use of simple trigonometry in aeronautics.

We have the formula for centrifugal force, $F = MV^2/R$. Let us assume that by

this formula we have found that in order for a plane weighing 1200 pounds to fly a circle of given radius at a given air speed has a centrifugal force of 692 pounds and hence requires a centripetal force of 692 pounds, in order to overcome the centrifugal force. Suppose we wished to determine the angle of bank necessary to supply this horizontal component of lift. Fig. 7 shows how simple trigonometry can be used in the solution of this problem.

The vector AC represents the upward force of the wings of the plane necessary to supply the sustentation AB and the centripetal force AD.

We have now of course the simple problem of determining the angle β such that its tangent is 692 divided by 1200.

The finding of the height of a cloud bank is illustrated in Fig. 8. To find the height of a cloud bank a searchlight at S shines straight up to the bottom of the cloud at C. By means of an instrument called a clinometer at some point P, the angle of elevation of C is determined. The height of the cloud bank, CS, is then determined, of course, by multiplying the distance PS by the tangent of the angle of elevation.

Fig. 9 shows the combination of rotation on forward motion of a propeller blade, and shows how any element of the propeller blade travels on a helix. The pitch of the propeller blade is the angle at A between the circle and helix shown in the figure when the forward motion is normal. Simple trigonometry is used to find the forward motion from the pitch of the propeller, or vice versa.

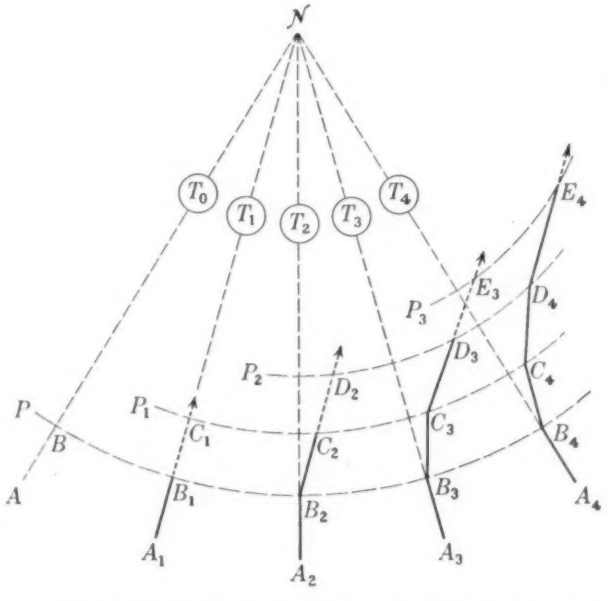

FIG. 10. Diagram illustrating the theory of the eastward deflection of northward moving air in the northern hemisphere.

DEFLECTION OF MOVING AIR MASSES

Let us skip, at this point, to one aspect

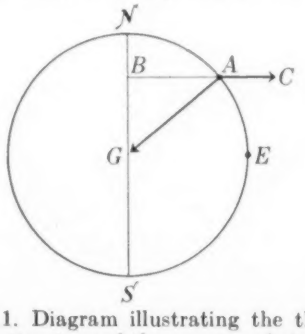

FIG. 11. Diagram illustrating the theory of the deflection toward the equator of air initially moving toward the east.

of meteorology, which is, of course, a very essential portion of the science of aeronautics.

It is interesting to learn why an air mass moving toward the pole tends to be deflected towards the east, and consequently to move in a circular path clockwise in the northern hemisphere. The reasons for this phenomena are shown in Fig. 10.

Stated very briefly, the explanation is that a body of air some distance from the pole has an appreciable eastward component of velocity due to the rotation of the surface of the earth toward the east. As the air mass proceeds northward, it tends to retain the eastward component of motion; that is, the distances BB_1, C_1C_2, D_2D_3, etc., tend to remain equal. Hence the path of the air mass veers to the right of the meridian.

In a similar manner it can be shown how air tending to move toward the equator is also deflected, but in this case toward the west.

Fig. 11 illustrates how the centrifugal force of rotation of the earth tends to force eastward moving air in the direction of the equator. The explanation is as follows.

If air is moving toward the east initially, it is rotating around the axis of the earth more rapidly than the surface of the earth is rotating—more rapidly than "stationary air." Therefore it tends to be thrown off from the axis of the earth in the direction of the arrow AC. We observe, however, that except at the equator this arrow does not point straight up—it slants toward the equator. This means that a component of the centrifugal force of the earth's rotation tends to force this air to move toward the equator. Hence in the northern hemisphere it veers to the right.

In a similar manner, it can be shown that westward moving air having less centrifugal force tends to be forced northward. Thus we see that both eastward and westward moving air in the northern hemisphere is veered to the right. This was true also of northward and southward moving air. Consequently, in the northern hemisphere air starting to move in any direction whatever tends to veer to the right. Similarly, in the southern hemisphere air moving in any direction tends to be deflected to the left.

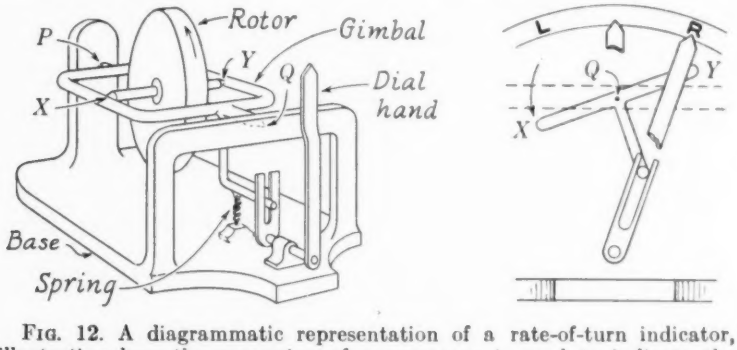

FIG. 12. A diagrammatic representation of a rate-of-turn indicator, illustrating how the precession of a gyroscope is used to indicate the amount of change in direction of flight.

We have space for a close-up view of but one more illustration of the application of science to aeronautics.

INSTRUMENTS FOR BLIND FLYING

It is interesting to learn how the principle of precession in a gyroscope is used in the instrument called the rate-of-turn indicator—one of the instruments used for blind flying. The rate-of-turn indicator indicates very precisely the amount of change of direction of the airplane.

You may ask, why is not the rate of turn of the airplane indicated by the compass? The answer is that because of

the banking necessary in a plane when making a turn, the compass does not correctly indicate the amount of turn by reason of what is known as the "northerly turning error" which there is not space here to explain. It has to do with the fact that the magnetic lines of force of the earth are not horizontal except at the equator.

Fig. 12 is the diagrammatic representation of the gyroscope mechanism of the rate-of-turn indicator. Note how the rotor, a small fly wheel, is mounted on an axle XY which in turn is mounted in a "gimbal" having an axle PQ mounted in the frame of the instrument. The instrument is set in the plane so that the axis PQ is parallel to the longitudinal axis of the plane, with P pointing forward.

Pitching the plane (moving its nose up or down) does not affect the rotor; the axis XY continues to point in the same direction, and the instrument merely rotates on this axis.

Banking the plane has no effect upon the rotor; the gimbal remains horizontal—the instrument itself merely rotates on axis PQ.

However, if the airplane changes direction, consequently changing the direction of axis PQ, the force of precession of the gyroscope tends to change the direction of axle XY. If P moves to the right, precession forces X downward. This moves the dial hand to the right as shown in the right-hand illustration, indicating a right turn. Similarly, forcing P to the right, causes the Y end of the axle XY to move down and the dial hand is moved to the left. The greater the speed of the turn the farther to the right or left the pointer moves.

Summary

In this article we have attempted to show how limitless are the applications of mathematics and science in the general field of aeronautics, and how many and varied are those applications which are suitable to be included in a high school course in aeronautics, or in mathematics or science.

We have selected a few of the more characteristic and representative samples for a closer examination.

It is to be hoped that teachers of mathematics and science throughout the country will realize that with the rapidly increasing activities and interest in aeronautics, there is to be found a powerful stimulus and motivation of students in the study of mathematics and science. We all realize how much easier it is for students to understand and remember facts and principles of mathematics and science when these are made vivid by application, and when the learning is strongly motivated by the learner's interest in the fields of application.

No teacher should presume that he or she must have had a course in aeronautics in order to bring many applications of this kind into the mathematics or science classroom. Indeed, many high schools throughout the country are beginning to feel that one of the most effective methods of teaching mathematics and science is to "take the bull by the horns" and give a course in aeronautics. Such a course will naturally treat in part the art of flying, about which the student can learn from the many books already printed on that subject, but will deal primarily with the application of mathematics and science to aeronautics.

The illustrations used in this article are taken from *Elements of Aeronautics* a high school text in aeronautics by Pope and Otis, copyright 1941 by World Book Company, Yonkers, New York.

The "Art of Nombryng"

By E. R. Sleight
Albion College, Albion, Michigan

The *Art of Nombryng* is a translation into 15th Century English of *de Arte Numerandi*, which is attributed to John of Holywood,[1] better known by the name of Sacrobosco. In this treatise nine operations are recognized—numeration, addition, subtraction, mediation, duplation, multiplication, division, progressions and extracting square and cube roots. The definitions of algorism, digits, articles, and composite numbers appear here in the same form and with the same meaning as in the **Craft of Nombryng*. The method of writing and reading numbers is thoroughly discussed. Understand that there are[2] .9. characters or figures that represent the .9. digits; they are .9.8.7.6.5-.4.3.2.1. The .10.[3] is called a cypher. It gives significance to the others; for without a cypher a pure article could not be written. Thus with these nine characters and the cypher all numbers may be written. Note well that all digits shall be written with one figure alone, and every article with a cypher, for every article is named for one of the digits as .10. for .1., .20. for .2. and so for the others. All digits shall be set in the first place and all articles in the second. Also all numbers from .10. to .100., which is excluded, must be written with .2. figures. And if it be an article, by a cypher first, and the figure written towards the left, that signifies the digit by which the article is named; if it be a compound number, first write the digit that is a part of the compound then write at the left side the article. All numbers that are from one hundred to one thousand which is excluded, shall be written with .3. figures, and all numbers from one thousand to ten thousand[4] must be written by .4. figures; and so forth. And understand that every figure in the first place signifies a digit, in the second place .10. times his digit, in the third place one hundred times, etc.

The methods used in addition and subtraction are the same as those used in the *Craft*, but with insistance upon form, as follows:

Addition—

Result	25
To whom it shall be added	13
The number to be added	12

Subtraction—

The remainder	12
Wherefrom we shall withdraw	25
The number to be withdrawn	13

The plan of procedure in multiplication is here indicated. In the first place we are told that there are 6 rules of multiplication. The first is a rule for multiplying a digit by a digit. As an example, to mutliply 4 by 8. Take the difference between 8 and 10 which is 2. Now write the article corresponding to 4 which is 40, and subtract 4×2. The result follows. The second case arises when a digit is multiplied by an article. Here we must multiply the digit by the digit for which the article is named. Then every unity shall stand for .10. and every article for .100. For example, to multiply 6×40. The product of 6 and 4 is first taken, in which product 2 is the article, and 4 the unit. According to

[1] Died 1244 or 1246.
[2] Note the use of the dot.
[3] Meaning the 10th character.
[4] Written .X.M.
* Mathematics Teacher, vol. XXXII, no.6.

the rule, the result may be written $2\times100+4\times10=240$. The remaining cases are multiplying a composite by a digit, article by an article, composite by an article, and composite by a composite. As in the *Craft of Nombryng*, the process of antry is used, which means that the operations proceed from left to right, except in case the multiplier is a digit. The form of the results would indicate that the order in case of a digit is reversed, as is shown by the following illustrations:

Resultans	1	1	0			1	3	2			7	3	6
Multiplicandus			5					4				3	2
Multiplicans		2	2				3	3			2	3	

The 5 standing above the last 2 indicates that the operation is from right to left, while in the last illustration, the form shows that the process is reversed.

Division appears in the *Art of Nombryng*. IN ORDER TO DIVIDE ONE NUMBER BY ANOTHER THERE ARE 2 NUMBERS PROPOSED. IT IS USED TO SEPARATE THE LARGER NUMBER INTO AS MANY PARTS AS THERE ARE UNITIES IN THE LESSER NUMBER. NOTE THAT IN THE OPERATION OF DIVISION, THERE ARE .3. NUMBERS, THE NUMBER TO BE DIVIDED, THE NUMBER DIVIDING, AND THE NUMBER RESULTING, OR QUOTIENT. THEREFORE IF THOU WILT ANY NUMBER DIVIDE, WRITE THE NUMBER TO BE DIVIDED OVER THE DIVISOR SO THAT THE LAST[5] OF THE DIVISOR BE UNDER THE LAST OF THE NUMBER TO BE DIVIDED, THE NEXT LAST UNDER THE NEXT LAST, AND SO WITH THE OTHERS, AS HERE INDICATED.

To be dyvydede	3	4	2
The dyvser	2	1	

BUT THERE ARE CASES IN WHICH THE LAST FIGURE OF THE DIVISOR MAY NOT BE SET UNDER THE LAST FIGURE OF THE DIVIDEND. This has reference to the case in which the last digit of the divisor is larger than the last digit of the dividend. When this is true then there is an antry of the digits of the divisor as is indicated in the first illustration below.

The Dyvydede	3	5	5	1	2	2		8	8	6	3	7	0	4
The Dyvyser		5	4	3				4	4	2	3			

THE ORDER BEING OBSERVED WE MUST WORK FROM THE LAST FIGURE OF THE DIVISOR AND SEE HOW OFT IT MAY BE WITHDRAWN FROM THE FIGURE (OR FIGURES) ABOVE ITS HEAD. THEN REMOVE THE RESULT THIS OFTEN. THIS BEING DONE WE MUST SET FORWARD (antry) THE FIGURES OF THE DIVISOR BY ONE DIFFERENCE TOWARDS THE RIGHT AS BEFORE OBTAINING THIS RESULT:

Dyvydede	2	9	3	2	2
Dyvyser		5	4	3	

Now performing the operation of removing .5. times the divisor from the dividend, and setting forward the digits in the divisor, .4. is obtained as the final digit in the quotient. The entire process is thus indicated in the text.

Quocient				6	5	4
Dyvydede	3	5	5	1	2	2
Dyvyser		5	4	3		

AND IF IT HAPPENS THAT AFTER SETTING FORWARD THE FIGURES OF THE DIVISOR THE RESULT MAY NOT BE WITHDRAWN FROM THE DIVIDEND THEN WE MUST SET A CYPHER FOR THE NUMBER IN THE QUOTIENT, THEN SET THE FIGURES OF THE DIVISOR FORWARD AS BEFORE, AND SO WE SHALL DO IN ALL NUMBERS TO BE DIVIDED, FOR WHERE THE DIVISOR MAY NOT BE WITHDRAWN WE MUST SET THERE A CYPHER.

One paragraph is devoted to progressions. A PROGRESSION IS AN AGGREGATION OF NUMBERS OF EQUAL EXCESSES, BEGINNING WITH ONE OR TWO. OF PROGRESSIONS ONE IS NATURAL OR CONTINUOUS, AND THE OTHER BROKEN OR DISCONTINUOUS. NAT-

[5] "Last" to the left, as is true in all translated parts of this article.

URAL IS WHEN WE BEGIN WITH ONE AND INCREASE BY ONE, AS .1.2.3.4.5.6., ETC. SO THAT THE NUMBER FOLLOWING PASSETH THE OTHER BEFORE BY .1. BROKEN IT IS WHEN WE LEAP FROM ONE NUMBER TO ANOTHER AND KEEP NOT THE CONTINUOUS ORDER AS .1.3.5.7., ETC. ALSO WE MAY BEGIN WITH 2, THUS, .2.4.6.8., ETC., AND THE NUMBER IN EACH CASE FOLLOWING PASSETH THE OTHER BY .2. AND NOTE WELL THAT NATURAL PROGRESSIONS ALWAYS BEGIN WITH ONE, BUT BROKEN PROGRESSIONS BEGIN WITH .1. OR WITH .2. OF THE NATURAL PROGRESSIONS THERE ARE[6] TWO RULES OF WHICH THE FIRST IS WHEN THE PROGRESSION ENDS IN AN EVEN NUMBER, BY THE HALF[7] THEREOF MULTIPLY THE NEXT LARGER NUMBER. EXAMPLE: .1.2.3.4. HERE WE MULTIPLY .5. BY .2. AND SO .10. COMETH, THAT IS THE TOTAL RESULT. THE SECOND RULE IS WHEN A NATURAL PROGRESSION ENDS IN AN ODD NUMBER. TAKE THE MIDDLE NUMBER AND MULTIPLY THIS BY THE LAST. EXAMPLE .1.2.3.4.5.6.7. HERE MULTIPLY .4 BY .7. AND THIS SHALL BE THE RESULT. OF BROKEN PROGRESSIONS THERE ARE ALSO TWO RULES; AND THE FIRST IS THIS: WHEN THE BROKEN PROGRESSION ENDS IN AN EVEN NUMBER BY THE HALF THEREOF MULTIPLY THE NEXT NUMBER TO THAT HALF AS .2.4.6.8. HERE MULTIPLY .4. BY .5. AND .20. IS THE SUM OF ALL THE PROGRESSION. THE SECOND RULE IS THIS: WHEN THE BROKEN PROGRESSION ENDS IN AN ODD NUMBER TAKE THE TOTAL (number) OF ALL THE ODD NUMBERS AND MULTIPLY BY HIMSELF AS .1.3.5.7. MULTIPLY .4. BY HIMSELF AND THE SUM OF ALL THESE NUMBERS WILL BE .16.

HERE FOLLOWETH THE EXTRACTION OF ROOTS, THE FIRST BEING SQUARE ROOTS. WHEREFORE WE SHALL SEE WHAT IS A SQUARE NUMBER AND WHAT IS THE ROOT OF A SQUARE NUMBER, AND WHAT IT IS TO DRAW OUT THE ROOT OF A NUMBER. AND ABOVE ALL NOTE THIS DIVISION: OF NUMBERS SOME ARE LINEAL, OTHERS SUPERFICIAL, OTHERS QUADRATES, AND OTHERS CUBIC OR SOLIDS. Here is brought out clearly that the lineal has one dimension, the superficial two while the solid has three. BECAUSE A NUMBER MAY BE MULTIPLIED BY ANOTHER IN .2. WAYS, THAT IS BY ITSELF OR BY ANOTHER, UNDERSTAND THAT IF IT IS BY ITSELF THE RESULT IS A QUADRATE. WHEREFORE IT IS SEEN THAT ALL QUADRATE NUMBERS ARE SUPERFICIAL, THE CONVERSE IS NOT TRUE. THE ROOT OF A QUADRATE IS THAT NUMBER WHICH FORMS THE NUMBER BY HIMSELF. THUS .4. IS THE FIRST QUADRATE NUMBER AND .2. IS ITS ROOT. A SOLID NUMBER IS ONE THAT COMES BY DOUBLE MULTIPLICATION, AND IT IS CLEAR THAT IT HAS .3. DIMENSIONS, LENGTH, BREADTH, AND THICKNESS. BUT A NUMBER MAY BE MULTIPLIED TWICE IN TWO WAYS, EITHER BY ITSELF OR BY OTHERS. IF THE NUMBER BE MULTIPLIED BY ITSELF TWICE, THE FIRST RESULT IS A QUADRATE, THE OTHER A CUBIC, AND IS A SOLID. IF THE NUMBERS ARE NOT ALIKE THE SOLID IS ELLIPTIC AND NOT CUBIC. WHEREFORE ALL CUBIC NUMBERS ARE SOLIDS BUT THE REVERSE IS NOT TRUE. IT IS HEREBY CLEAR THAT NO NUMBER IS THE ROOT OF A QUADRATE AND A CUBIC. THEREFORE SINCE MULTIPLYING UNITY BY HIMSELF ONCE OR TWICE GIVES UNITY, THEN UNITY IS NOT A NUMBER.

NOW UNDERSTAND THAT BETWEEN EVERY .2. QUADRATES THERE IS A MEAN PROPORTIONAL. IT IS OBTAINED THUS: MULTIPLY THE ROOT OF THE ONE QUADRATE INTO THE ROOT OF THE OTHER AND THE RESULT WILL SHOW THE MEAN.

TO EXTRACT THE ROOT OF A QUADRATE NUMBER MEANS TO SEE WHETHER THE PROPOSED NUMBER BE A QUADRATE. AND IF IT IS NOT A QUADRATE, THEN FIND THE ROOT OF THE LARGEST QUADRATE LESS THAN THE NUMBER PROPOSED. THEREFORE IF THOU WILT FIND THE ROOT OF ANY QUADRATE WRITE THE NUMBER AND COUNT THE NUMBER OF FIGURES, TO DETERMINE IF IT (the number of figures) BE EVEN OR ODD. IF IT IS EVEN, THEN THE WORK MUST BE BEGUN WITH THE LAST SAVE ONE, BUT IF ODD THEN WITH THE LAST. OR TO SAY IT A SHORTER WAY ALWAYS BEGIN WITH THE LAST[8] ODD

[6] Rules for Finding Sum.
[7] Half Sum.
[8] This refers to position, not to value.

DIGIT. WE MUST THEN FIND A DIGIT, WHICH SQUARED IS EQUAL TO THIS LAST, OR AS NEAR EQUAL AS POSSIBLE. In case the number of digits is even, this means that the square must be equal or less than the number represented by the last two digits. If the number is odd, then it must be equal to or less than the last digit only. We are then instructed to place the square in its proper place, AND WITHDRAW IT FROM THAT WHICH IS OVER IT. WE MUST THEN DOUBLE THE DIGIT AND SET THE DOUBLE UNDER THE NEXT FIGURE TOWARD THE RIGHT HAND. The second digit in the answer is found by division, following which WE MULTIPLY THE DOUBLE BY THE SECOND FIGURE AND ADD AFTER IT THE SQUARE OF THE SECOND FIGURE, THEN SUBTRACT.

As an illustration 15139 is given. This is not a perfect square, but the solution leaves no residue so I assume that an error has been made, all of the work indicating that the number should have been 15129. The process step by step is as follows: The number of digits being odd, the process first considers the digit 1. Obviously the first digit in the answer is 1. Squaring this and removing the result the number reduces to 5129. Now take two times the 1, and place the result under the next digit or 5, and the problem takes this form: $\frac{5129}{2}$. By division 2 is found to be the second digit in the answer. Multiplying the double (2) by the second figure (2) and adding the square of the second (4) we must subtract 44 from 51 and there remains 739, and the first two digits in the answer are 1 and 2. Doubling this answer and placing it according to instructions there results $\frac{729}{24}$. Division gives 3 for the final digit in the answer. If to 3 times 26, 9 is annexed, the result is 729, showing that 15129 is a quadrate number. The form used for indicating the above process is here given.

Number	1	5	1	2	9
The double		2		4	
The result	1		2		3

It will be observed that in rather an indirect way the method now in use is here outlined. However, it was not until the middle of the 16th century when Robert Record's Arithmetic was published did the method of today appear in its entirety, and to him is given the credit for having first used it. The author completes the explanation concerning square roots thus: THAT DONE, EITHER OUGHT OR NOUGHT WILL REMAIN. IF NOUGHT, THEN IT SHOWETH THAT THE PROPOSED NUMBER WAS A QUADRATE. AND IF OUGHT REMAINETH THAT SHOWETH THAT THE NUMBER PROPOSED WAS NOT A QUADRATE. IF THOU WILT PROVE THE WORK, MULTIPLY THE LAST RESULT BY ITSELF AND THOU SHALT FIND THE SAME NUMBER THAT THOU HADDEST BEFORE. BUT IF THOU HAST A RESIDUE, THEN WITH THE ADDITION THEREOF OF THIS RESIDUE, THOU SHALT FIND THE FIRST NUMBER AS BEFORE.

HERE FOLLOWETH THE FINDING OF ROOTS OF CUBIC NUMBERS. WHEREFORE WE MUST SEE WHAT A CUBIC NUMBER IS, WHAT IS ITS ROOT, AND WHAT IS MEANT BY THE EXTRACTION OF A ROOT. AS BEFORE SAID, A CUBIC NUMBER IS ONE THAT COMES BY MULTIPLYING A NUMBER TWICE BY ITSELF, OR ONCE BY ITS QUADRATE. THE ROOT OF A CUBIC NUMBER IS A NUMBER THAT, MULTIPLIED TWICE BY ITSELF OR ONCE BY ITS QUADRATE, WILL GIVE THE NUMBER. TO FIND THE ROOT OF A CUBIC, IT IS FIRST NECESSARY TO FIND IF THE PROPOSED NUMBER IS A CUBIC. IF NOT THEN ONE MUST EXTRACT THE ROOT OF THE LARGEST CUBIC LESS THAN THE PROPOSED NUMBER WHOSE ROOT IS TO BE FOUND. FIRST MARK OFF THE PLACES IN THREES. AND WITH THE LAST THREE PLACES, A DIGIT WHICH CUBED MUST DO AWAY WITH THAT PLACE, OR AS NEAR IT AS POSSIBLE. Here as in square root the process is very similar to the process now used—first subtract the proposed cube, then find the trial divisor, the two corrections, etc. The writer finishes the discussion of cube roots by calling attention to the fact that IF NOTHING REMAINS IN THE PROCESS, THEN

THE NUMBER PROPOSED IS A PERFECT CUBE BUT IF A RESIDUE REMAINS, THEN IT IS NOT.

To aid in determining the squares and cubes of numbers tables were formed as follows: in each case, the numbers in the principal diagonal represent the required results.

1	2	3	4	5	6	7	8	9
2	8	12	16	20	24	28	32	36
3	18	27	36	45	54	63	72	81
4	32	48	64	80	96	112	128	144
5	50	75	100	125	150	175	200	225
6	72	102	144	180	216	252	288	324
7	92	147	196	245	294	343	392	441
8	128	192	256	320	384	448	512	576
9	162	243	324	405	486	576	648	729

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

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THE ALGEBRA OF OMAR KHAYYAM

By DAOUD S. KASIR, Ph.D.

The author of the *Rubaiyat* was also an astronomer and mathematician. This work presents for the first time in English a translation of his algebra. In the introduction, Mr. Kasir traces the influence of earlier Greek and Arab achievements in mathematics upon the algebra of Omar Khayyam and in turn the influence of his work upon mathematics in Persia in the Middle Ages. The translation is divided into chapters, and each section is followed by bibliographical and explanatory notes.

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Measurement in Arithmetic Textbooks for Grade Three

By AGNES G. GUNDERSON
University of Wyoming, Laramie, Wyoming

THIS STUDY was undertaken in an attempt to discover the amount and kind of measurement contained in arithmetic texts for Grade Three. It was felt that such information would be of help in planning the course of study in arithmetic for Grade Two. In other words it is an endeavor to find at least a partial answer to the question "What knowledge of measures should the child have upon entering the third grade?"

Twelve arithmetic texts of recent copyright were analyzed to find the amount of space devoted to the study of the following measures: linear, liquid, dry, weight, and number. The books used were the following:

1. *Champion Arithmetics* Stepped-Up Edition, Grade Three, by Joseph C. Brown, Helen Cook Mirick, J. Freeman Guy, and Albert C. Eldredge. Row, Peterson and Company, Evanston, Ill. 1937.
2. *Child-Life Arithmetics* Grade Three, by Clifford Woody, Frederick S. Breed, and James R. Overman. Lyons and Carnahan, Chicago. 1936.
3. *Daily-Life Arithmetics* Grade Three, by Guy T. Buswell, William A. Brownell, and Lenore John. Ginn and Company, Chicago. 1938.
4. *Iroquois New Standard Arithmetics* Grade Three, by Harry De W. De Groat, and William E. Young. Iroquois Publishing Company Inc., Syracuse, New York. 1938.
5. *Jones-Wheat Arithmetics* Book One, by Robert L. Jones and Harry G. Wheat. D. C. Heath and Company, Chicago. 1935.
6. *The Master Key Arithmetic* Grade Three, by Frank L. Clapp, Harriet Sleeper, Joy Mahachek and Lillian Lamb Ralya. Houghton Mifflin Company, Chicago. 1937.
7. *Mastery Arithmetic* Grade III, by George R. Bodley, Charles S. Gibson, Ina M. Hayes, Bruce M. Watson. D. C. Heath and Company, Chicago. 1934.
8. *Modern School Arithmetic* New Edition, Third Grade, by John R. Clark, Arthur S. Otis, and Caroline Hatton, Edited by Raleigh Scharling. World Book Company, Yonkers-on-Hudson, New York. 1937.
9. *The New Curriculum Arithmetics* Grade Three, by Leo J. Brueckner, C. J. Anderson, G. O. Banting and Elda L. Merton. The John C. Winston Company, Chicago. 1939.
10. *The New Trend Arithmetic* Third Year, by Harry O. Gillet, Thomas J. Durell, and Fletcher Durell. The Charles E. Merrill Company, Chicago. 1936.
11. *Social Utility Arithmetics* Book One, by George D. Strayer and Clifford B. Upton. American Book Company, Chicago. 1937.
12. *Study Arithmetics* Grade Three, by F. B. Knight, G. M. Ruch, J. W. Studebaker, W. C. Findley and William S. Gray. Scott, Foresman and Company, Chicago. 1934.

All the texts analyzed include more or less material in measurement. The writer found that the authors of the various texts do not include in the index every occurrence of units of measure. It is probable that where the author lists certain content in the index under the topics: *Measures, dozen, pound, foot,* etc., his aim is to develop an understanding of, and an acquaintance with units of measure and the measuring system as well as to give practice in computation. The following problems are illustrative:

"Mother asked for one pound of beef. The butcher cut a piece that weighed 15 ounces Did he cut too much or too little? How much?"[1]

"The milkman brings 2 qt. of milk every day. In 8 days he brings _____ qts. or _____ gal."[2]

"Try to draw a straight line 2 inches long on your paper without using your ruler. Measure the line. Is it longer or shorter than 2 inches?"[3]

Where indexing does not occur the em-

[1] *The New Curriculum Arithmetics,* pp. 213.
[2] *Daily-Life Arithmetics,* pp. 259.
[3] *Study Arithmetics,* pp. 183.

TABLE I

Kind of Measure	Unit of Measure	Amount of Space (Pages) Devoted to Measurement											
		Champion Arith. 253 pages	Child-Life Arith. 238 pages	Daily Life Arith. 277 pages	Iroquois Arith. 208 pages	Jones-Wheat Arith. 337 pages	Master Key Arith. 234 pages	Mastery Arith. 176 pages	Modern School Arith. 272 pages	New Curriculum Arith. 274 pages	New Trend Arith. 300 pages	Social-Utility Arith. 279 pages	Study Arith. 307 pages	Total (12 books) 3,153 pages
Linear	block (city)	.73	—	.1	—	.35	.1	.9	—	.1	.37	—	.03	2.68
	inch, foot (children's height)	.2*	1.0*	.47*	.75	.3	3.34	—	.24	—	—	.32	.07	5.12
		—	—	—	—	.22*	1.7*	—	.05*	1.17*	.1	.15*	—	4.96*
	inch, foot, yard	3.35	.2	1.3	4.68	3.68	2.60	1.79	7.36	.64	1.0*	.86	4.93	32.20
		4.25*	3.2*	6.11*	2.00*	.74*	5.73*	4.2*	1.75*	7.23*	.81	4.0*	4.0*	44.21*
	Inch, foot, yard, rod, mile	—	—	—	—	.63	—	—	1.01	—	—	—	—	1.64
		—	—	—	—	4.32*	—	—	—	1.0*	—	—	—	5.32*
	mile	3.96	3.5	5.35	—	3.17	4.71	1.4	—	—	2.85	3.23	2.26	30.43
		—	—	.3*	—	1.0*	—	—	—	1.2*	—	—	—	2.5*
Liquid	gill	—	—	—	—	.03	—	—	—	—	—	—	—	.03
	glass	.03	—	2.4	.2	.1	—	—	.1	—	—	—	.14	2.97
	quart, gallon (gasoline, oil)	.3	—	—	—	.7	.1	.1	—	.17	—	—	—	1.37
		.5*	—	—	—	—	—	—	—	—	—	—	—	.5*
	cup, pint, quart, gallon	1.94	1.5	.8	2.14	2.82	.23	1.92	1.84	.32	.2	1.89	1.70	17.3
		1.65*	2.4*	2.0*	.8*	1.35*	.1*	1.88*	1.20*	4.65*	1.0*	1.93*	1.78*	20.74*
	can	.03	—	.2	.21	2.66	—	—	.34	.17	.39	.5	.44	4.94
	half-pint	—	—	—	—	—	—	.28	—	—	—	—	—	.28
Dry	pint, quart, peck, bushel	.7	1.11	.22	1.42	5.58	—	.39	.7	—	.3	.1	.1	10.62
		1.03*	—	1.65*	.5*	2.1*	—	1.0*	.15*	—	—	—	—	6.43*
Weight	pound (children's weight)	1.49	3.2	.18	—	.75	.5	—	.08	.48	1.75	1.33	1.0*	9.89
		—	—	—	—	.27*	1.7*	—	1.0*	1.7*	—	.1*	.13	5.77*
	ounce, pound	1.58	.21	1.37	4.31	3.09	.18	1.07	3.03	.14	2.05	.55	1.1	18.68
		1.3*	.6*	1.0*	1.0*	.2*	—	.84*	—	1.15*	1.0*	.93*	1.18*	9.20*
	ounce, pound, ton	—	—	—	—	.4	—	—	.3	1.0*	—	—	—	.7
		—	—	—	—	1.3*	—	—	1.1*	—	—	—	—	3.4*
	ton	—	—	—	.1	.41	—	.03	—	—	—	—	—	.54
		—	—	—	—	.33*	—	—	—	—	—	—	—	.33*
Number	dozen	3.93	.1	1.5*	.46	2.28	.23	.63	1.3	.8	1.96	1.75	1.95	15.39
		.25*	2.5*	—	1.00*	—	1.24*	1.0*	—	.97*	1.9*	1.3*	.05*	11.71*
	dozen, gross	—	—	—	—	—	—	—	.1	—	—	—	—	.1
		—	—	—	—	1.08*	—	—	.1*	—	—	—	—	1.18*
	pair	.08	.26	.11	1.46	1.91	—	.18	1.93	.03	—	—	—	8.71
		—	—	—	—	—	—	—	—	.59*	1.67	.64	.44	.59*
Total		18.12	10.08	12.03	15.73	28.86	11.99	8.69	18.33	2.85	12.45	11.17	13.29	163.59
		9.18*	9.7*	13.03*	5.30*	12.91*	10.47*	8.92*	5.35*	20.66*	4.9*	8.41*	8.01*	116.84*
		27.30	19.78	25.06	21.03	41.77	22.46	17.61	23.68	23.51	17.35	19.58	21.30	280.43

* Listed in index.

TABLE II

Per cent of Content Devoted to Measurement

Kind of Measure	Champion Arithmetic 253 pages	Child-Life Arithmetic 238 pages	Daily-Life Arithmetic 277 pages	Iroquois Arithmetic 208 pages	Jones-Wheat Arithmetic 337 pages	Master Key Arithmetic 234 pages	Mastery Arithmetic 176 pages	Modern-School Arithmetic 272 pages	New Curriculum Arithmetic 274 pages	New Trend Arithmetic 300 pages	Social-Utility Arithmetic 279 pages	Study Arithmetic 307 pages	Average of 12 books 3,155 pages
Linear	3.2	1.5	2.4	2.6	2.4	4.6	2.3	3.2	.3	1.4	1.6	2.4	2.3
	1.7*	1.8*	2.5*	1.0*	1.9*	3.2*	2.4*	.6*	3.8*	.3*	1.5*	1.3*	1.8*
Liquid	.9	.6	1.2	1.2	1.9	.2	1.3	.8	.2	.2	.9	.7	.8
	.9*	1.0*	.7*	.4*	.4*	—	1.1*	.5*	1.7*	.3*	.7*	.6*	.7*
Dry	.3	.5	.1	.7	1.7	—	.2	.3	—	.1	.04	.03	.3
	.4*	—	.6*	.2*	.6*	—	.6*	—	—	—	—	—	.2*
Weight	1.2	1.4	.5	2.1	1.3	.3	.6	1.2	.2	1.3	.6	.4	.9
	.5*	.3*	.4*	.5*	.6*	.7*	.5*	.8*	1.4*	.3*	.4*	.7*	.6*
Number	1.6	.1	.6*	.9	1.2	.1	.4	1.3	.3	1.2	.8	.8	.8
	.1*	1.1*	—	.5*	.3*	.5*	.6*	—	.6*	.6*	.5*	—	.4*
Total	7.2	4.1	4.2	7.5	8.5	5.2	4.8	6.8	1.0	4.2	3.9	4.3	5.1
	3.6*	4.2*	4.8*	2.6*	3.8*	4.4*	5.2*	1.9*	7.5*	1.5*	3.1*	2.6*	3.7*
	10.8	8.3	9.0	10.1	12.3	9.6	10.0	8.7	8.5	5.7	7.0	6.9	8.8

* Listed in index.

phasis is on the computational phase or, in a page of miscellaneous problems only one or a very few may deal with measures. The following illustrate:

"Find the cost of 2 pounds of coffee at 26 cents a pound."[4]

"If one dozen eggs costs 18¢ how much will two dozen cost?"[5]

Since it was felt that a knowledge of the vocabulary is part of the arithmetic of measures all those problems or parts of problems using such vocabulary are included in the measurement count of the books analyzed whether or not they are listed in the index. Separate tabulations, however, were made for indexed and non-indexed content.

Table I shows the amount of space in pages given to measurement in the various text books Table II shows the proportionate amount of measurement content in these same books.

The amount of space given to measurement ranges from 5.7% in one book to 12.3% in another with the median at 8.8%.

With one exception all the books analyzed give considerably more space to linear measure than to any other; linear measure alone occupying one-third or more of the total measurement content in eleven of the twelve books analyzed. In ten books out of the twelve, less space is given to *dry* than to any other kind of measure. Four texts give less than one-tenth per cent of their content to dry measure.

Ten books include problems of distance computed in miles to the extent of 32.9 pages while only 2.7 pages (in eight books) are given to problems using blocks (city blocks) as the measuring stick.

Eleven books gave a total of 10 pages to measuring children's heights with *inch* and *foot* as the units of measure. The *inch*, *foot*, *yard*, were used as measuring sticks for the following commodities: cloth, ribbon, thread, yarn, lace, carpet, clothesline, wire, fish, sidewalk, fence, doll, board, rope, paper.

[4] *New Trend Arithmetic*, pp. 227.
[5] *Study Arithmetics*, pp. 266.

TABLE III

Amount of Space (Pages) Given to Miscellaneous Measures

	Champion Arithmetic	Child-Life Arithmetic	Daily-Life Arithmetic	Iroquois Arithmetic	Jones-Wheat Arithmetic	Master-Key Arithmetic	Mastery Arithmetic	Modern School Arithmetic	New Curriculum Arithmetic	New Trend Arithmetic	Social Utility Arithmetic	Study Arithmetic	Total (12 books)
Bar	.59	.25	.09	.45	1.14	.2	.23	.25	.07	1.33	.21	—	4.81
Basket	—	—	—	—	.3	.5	—	—	—	.1	—	.1	1.0
Box	—	.58	.2	.15	.86	.07	4.63	.16	—	.1	.1	.1	6.95
Brick	.1	.13	—	—	.03	—	—	—	—	—	—	—	.26
Bunch	.5	.75	.6	.57	.76	—	—	—	—	.03	.8	—	4.01
Cake	—	—	—	—	.1	—	—	.13	—	.15	—	.24	.62
Cone	.16	—	.09	—	—	.26	.32	.13	.17	.6	.5	.25	2.48
Crate	.1	—	—	.1	1.05	—	.1	—	—	—	—	—	1.35
Ear (corn)	—	—	—	—	—	—	.08	—	—	—	—	.78	.86
Head (lettuce)	—	—	—	—	.1	—	.16	—	—	—	.2	—	.46
Loaf (bread)	.33	.05	—	—	.2	—	3.5	—	.27	1.1	.51	—	5.96
Package	—	—	.1	.4	.1	.96	.61	.2	—	—	—	.1	2.47
Sack	.2	—	—	—	.3	—	—	—	—	—	—	.25	.75
	1.98	1.76	1.08	1.67	4.94	1.99	9.63	.87	.51	3.41	2.32	1.82	31.98

Problems of buying or selling gasoline and oil by the gallon and quart respectively are found in five books with a total of 1.9 pages. The *pint*, *quart*, *gallon*, were used to measure: milk, ice cream, lemonade, water, syrup, tomatoes, cider, paint, grape juice, vinegar, and molasses. The *gill* as a unit of measure was found in only one book and there only briefly mentioned. *Can* was used in nine books in problems dealing with: soup, tomatoes, corn, peas, peaches, salmon, fruit, vegetables, milk, and beans.

A large number of articles was measured by the *dozen:* onions, pencils, cakes, cookies, roses, oranges, eggs, rolls, bulbs, popcorn balls, apples, doughnuts, ears of corn, bananas, pears, buttons, napkins, candles, children, lemons, buns, plant, flowers, hats, toys, handkerchiefs, cans, candy bars, loaves of bread. *Pair* was used for measuring: mittens, scissors, shoes, gloves, skates, toys, boots, stockings, wheels, overshoes, hose and rubbers.

Dry measure was used for these products: beans, corn, berries, nuts, cherries, peas, wheat, oats, potatoes, plums, peaches, peanuts, pears, sugar, apples.

Ten books gave a total of 15.6 pages to problems on children's weights with *pound* as the unit of measure. The following articles were measured by *pound* and *ounce:* mail, letters, newspapers, books, meat, squash, butter, sugar, nuts, calves, carrots, apples, beans, water, pepper, milk grapes, cheese, peaches, cookies, tea, melons, coffee, bananas, candy, dates, tomatoes, duck, dog, cat, hog, candles, chalk, rock, box, salt, rice, coal, onions, cabbage and flour.

Table III gives a list of indefinite or miscellaneous measures used in the various books None of these items were indexed. Some terms are common to several books, e.g. *bar* is used in eleven books in problems with these articles: candy, chocolate, soap, ice cream, maple sugar. *Box* is used in ten books with the following commodities: cocoa, crayons, paints, cereal, crackers, cookies, berries, chalk, oranges, strawberries, apples, cornflakes. Nine books use *cone* in problems about ice cream. These terms are used in seven books: *bunch* for carrots, celery, radishes, sweet peas, grapes, onions, beets flowers; *loaf* for bread; *package* for seeds, drawing paper, and breakfast foods.

The following are mentioned only once and therefore not included in any of the tables: *3 for 5¢* (lemons), *team* (horses), *stories* (height of building), *barrel* (flour, apples), *basket* (apples), *bale* (cotton), *scoop* (ice cream), *load* (hay), *tablet* (paper), *cwt. score;* and these which were

indexed: *teaspoon* (vanilla), *tablespoon* (butter).

Summary

On the basis of this study the average arithmetic text for Grade Three contains 263 pages. Of this amount 23.4 pages, or 8.8% of the entire content is devoted to the study of measures divided as follows:

Linear measure, 10.8 pages or 4.1 per cent of entire book content.
Liquid measure, 4.0 pages or 1.5 per cent of entire book content.
Dry measure, 1.4 pages or .5 per cent of entire book content.
Weight measure, 4.1 pages or 1.5 per cent of entire book content.
Number measure, 3.1 pages or 1.2 per cent of entire book content.

If one includes under measures only that part of content that is indexed as such the amount is greatly reduced. The resulting amounts are: the average third grade arithmetic text contains 263 pages (same as above), of this amount 9.7 pages or 3.7% of entire book content is given to measurement classified as follows:

Linear measure, 4.8 pages or 1.8 per cent of entire book content.
Liquid measure, 1.8 pages or .7 per cent of entire book content.
Dry measure, .5 pages or .2 per cent of entire book content.
Weight measure, 1.5 pages or .6 per cent of entire book content.
Number measure, 1.1 pages or .4 per cent of entire book content.

This study shows the vocabulary and units of measure the child is going to encounter in his arithmetic text in Grade III. The teacher in Grade II can then include as many as practical of these in the informal arithmetic given the second year in order that the child may have a long learning period in which to become familiar with these measuring sticks before he is required to work problems dealing with them.

The study adds further testimony to the value and importance of such generally accepted phases of primary arithmetic as playing store and measuring in construction work.

It is a question if textbooks alone provide a sufficient amount of experience in using the various measuring sticks. Perhaps we can rightfully take it for granted that the teacher in making assignments will supplement the textbook, but even so one cannot over-emphasize the benefit to the child if in Grades I and II through such arithmetic activities as playing store and measuring he becomes familiar with the different units of measure and the vocabulary involved. Such experience is indispensable as foundation for the work in measuring of Grade III.

If problems in measurement are to be more than abstract computation and have meaning for the child, he must have gained through manifold experiences the concepts of these units of measure. The following might well be included as essential classroom equipment for Grades Two and Three: (1) household scales, (2) milk bottles of one-fourth pint, one-half pint, pint, and quart capacity as well as the tin measures ordinarily used for liquid and dry measures, (3) the various measuring sticks for linear measure; ruler, yardstick, tape measure, carpenter's rule, steel tape.

Notice to Subscribers!

Mr. Hermann J. Jordan has a class in mathematics and physics at the Weather School, Maxwell Field, Alabama. Many articles of The Mathematics Teacher have been of interest to the enlisted men there as well as those in the Weather Station. Will any subscribers who have back numbers of the Teacher that they think will be of interest to the above persons, kindly forward them *gratis* to Mr. Jordan.—The Editor.

Individual Differences and Course Revision in Plane Geometry

By JAMES M. LYNCH
South River, New Jersey

FEW TOPICS are of more concern to geometry teachers at the present time than the changed character of the student body that has come to them during the past ten years. Traditional college-preparatory students no longer represent a majority of the high school enrollment. Instead there is a preponderance of those students who, in the days prior to the depression, would have dropped out of school.

As these "new pupils" increased in number, the fact that they could not cope with the academic work of the conventional program began to stand out quite prominently. They failed miserably with plane geometry as Euclid had formulated it. The one thing that seemed certain about them was that abstract concepts, formulae, rigid demonstrations, etc. were completely beyond their grasp. Moreover, the danger, clearly sensed even from the first, that their presence might lower the standards of the traditional-course group loomed larger than ever.

In other words, the coming of the rank and file to high school actually brought the problem of individual differences, which heretofore existed mainly in words in educational lectures, courses, and periodicals down to earth and into the classroom. The task of adequately accommodating the unselected many served to emphasize the significance of the problem in concrete situations, and made concrete attacks upon it inevitable. Eventually, it gave definite impetus to a movement toward radical revision of the course of study.

Outstanding among the many modes of attack that came rapidly to the fore, and one that apparently disposed of the difficulty satisfactorily, took its point of departure from one of Goethe's maxims: "The greatest art in theoretical and practical life consists in changing a *problem* into a *postulate*." Accordingly, following this procedure, it was urged that if the pupils' capacity for learning the congruence theorems, for example, is low, do not *try* to teach those theorems—just postulate them, that is, treat them as axiomatic, as data self-evident immediately and requiring no proof. Is the class too dull to understand the proposition: if two straight lines are cut by a transversal, and the alternate interior angles are equal, the lines are parallel? Then postulate that proposition, too, and all annoyances and confusions about how to "get it across" will vanish as though blown away on the wings of a gentle breeze.

Merely turning the problems of teaching plane geometry to the "masses" into postulates has the rather attractive advantage of being a very pleasantly painless procedure. It has the added advantage of increasingly gaining favor and support, as a teaching device, among professors of education, curriculum experts, and textbook authors. Indeed, one of the distinctive marks of a modern progressive text is the number of postulate assumptions and assumed theorems it contains—the greater the amount of postulation, the more "modern" and "progressive" it is.

An equally popular and somewhat similar attempt to come to terms with the problem of varying abilities took the form of what is known as the "*A*, *B*, and *C* theorems" or the "starred and double-starred exercises" technique. Here, too, the idea is to eliminate the proofs of difficult theorems without, however, actually omitting them from the course.

In employing the "*ABC*" technique, the work is arranged on three levels of difficulty. Pupils in the lower I.Q. brackets

prove formally only the theorems marked "*C*." For the average-ability pupils, the course includes those theorems designated by the letter "*B*." Theorems labeled "*A*" serve as a preparation for the college entrance examinations, and may be assigned to the brightest pupils. Thus, the ablest pupils are challenged without in any way discouraging the less gifted.

Starring exercises is only another way of accomplishing the same result—or marking out separate courses for the superior, average, and slow classes. A small group, comprised of students who are planning to enter higher institutions or who show unusual aptitude in mathematics, attains a thorough mastery of the subject. By means of omitting material indicated by two stars, a course is provided for the less capable group. Both double-starred and single-starred work may be omitted to suit those pupils who range from slight proficiency to an absolutely hopeless state. Some of the latter might not get much beyond the construction of church-window designs, linoleum and quilt patterns, and fancy monograms, or pre-occupation with photographs of snow crystals, stream-lined trains, and the George Washington bridge. Nevertheless, they are happier in their work, and are stimulated to greater efforts than they would have been with the type of course offered in the past.

Unquestionably, such radical innovations do reduce school failures to a great extent—especially that "complete frustration" type of failure which so easily gives rise to feelings of inferiority, and encourages the development of anti-social attitudes. In so far as taking into account a much greater range of capacities than did the traditional course, operating to relieve discontent, bringing about greater interest and effort on the part of a greater number of students, and at the same time preserving high standards are concerned, they certainly may well lay claim to success in providing for individual differences. Indeed, it is undoubtedly their high degree of relevance to the problem of individual differences that made them so widely acceptable in so short a period of time.

On the other hand, the fact that the expression "individual differences" is a coat of many colors invites attention to the particular interpretation underlying the type of provision afforded by the utilization of postulation or *ABC* theorems or starred exercises or by any combination of one or more of these techniques.

From the brief summaries given above, it is clear that the chief purpose of these newer devices is to make the work of the course in plane geoemtry fit the pupils *as they are* in all their human variety when they come to class, and that elimination (of difficult proofs and other materials), in one way or another, plays the dominant role in achieving that purpose. Out of this recognition of the wide range of variability among pupils, and out of a strong faith in elimination as the machinery through which course adjustment or adaptation is best realized, there emerges a basic conception which, in more rsepects than one, harks back to the early days of mental testing. Statistics scientifically gathered by experimental psychology at that time seemed to lend support to the view which held: (1) that learning capacity is an innate trait; and (2) that it is practically unmodified by experience or training. For education this means that: the course provided for each individual should be determined by his native capacity; the school can do nothing to improve him; the teacher's job is more or less that of an umpire or referee—he or she just keeps score.

Though the fact appears to have received little notice, proponents of course revision through elimination, it seems, imply more than a bare endorsement of this fatalistic conception. While the program they offer attaches much importance to the differences that exist between individuals, that importance is attached primarily to the limitations of pupils. Where the old traditional course set out to *make* or *break* the learner, the newer revised course, by insisting that every pupil be allowed to

work up to the limits of his own capacity, seeks to insure that the subject-matter will not break even the most incapable; but it is not at all enthusiastic about the *making* phase of teaching. Its psychology sees individual differences as *limits*, and argues for educational restriction.

Subsequent psychology is more optimistic. There is a marked tendency to get away from the emphasis on heredity in general, and on original nature as an unalterable limitation in particular. "Everything is possible for everybody," according to Adler, "obstacles can be overcome; one must only know them." The whole tenor of the modern attitude may well be introduced by this remark of Adler's. The conclusion is, in the words of Powers and Uhl, that

> Anyone who can learn to count objects probably can learn to use symbols or counters for the objects, as in arithmetic and algebra. Anyone who can learn that water ordinarily runs downhill can learn something of gravity. Therefore, when it is asserted that a pupil can not learn a subject, what is meant is that he can not learn the subject under existing conditions.[1]

More recently, the educational psychology stemming from the experimental work of the present-day Gestalt school holds that achievement is only partly, not exclusively, conditioned by inheritance—opportunity and proper stimulation are also controlling factors.

Hence, a recognition of individual differences which merely identifies them and accepts variations among pupils as *limitations* is in no better position than was the old, rigid, inelastic course of tradition. The traditional course was in error in assuming that all individuals were alike, and that failure to learn resulted primarily from lack of application, stubbornness, and disobedience. Proponents of course revision through the elimination and omission of subject-matter, however, even though their work is thoroughly motivated by the recognition of variability among pupils, are equally at fault in unhesitatingly and confidently taking for granted that it is only the learners who differ from one another, and that there is only one method of teaching and learning. Both types of course take the pupils *as they are*, at the beginning of the course, and leave a very large percentage of them *as they were*; there is no conscientious effort to raise the great majority to higher levels of thinking and reasoning.

What is needed, especially in a democratic system of education, is a broadening and widening of the term "individual differences," in accordance with the concept of modern psychology, to include differences in rate of learning, differences in methods of learning, differences in materials, in books, and in the authors of books —as well as among the learners. A recognition, in short, that it is not only the subject-matter, but also the way in which it is presented that counts.

That the mode of presentation, of authors for example, is an important factor in understanding is a well-known fact which Stuart Chase a few years ago made the subject of his best seller, *The Tyranny of Words*. I. A. Richards, a psychologist and an advocate of Basic English, pointed out recently that the same piece of writing seldom receives the same response from any two readers. Many teachers themselves have had the experience of being able to learn more about Dewey's educational philosophy from Professor Horne's or some other exposition than from the original because of Dewey's style of presentation. Thorndike's educational psychology was a real tyranny of words in many a summer school and extension course until Gates' presentation came from the press. The subject-matter in Gates was the same —only the method of presentation was different. An interesting example, taken from the field of solid geometry, of how a change in mode of presentation served to make clear the theorem, "If two planes

[1] Powers, F. F. and Uhl, W. L., *Psychological Principles of Learning* (New York: The Century Company, 1933), p. 417.

intersect, their intersection is a straight line,"—a theorem which, by the way, is omitted in many of the present-day texts was published in a recent issue of this journal.[2] After the pupils had attempted to make drawings of various solid objects in the classroom, a short course in drawing

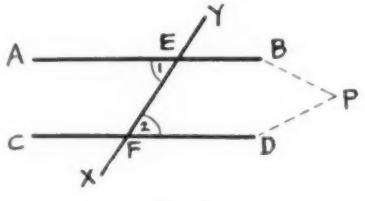

Fig. 1

was given. Then the theorem was assigned. The pupil was required, first, to make a model of the figure; second, to draw the figure on the board; and finally, to demonstrate the proposition to the class, using both the drawing on the board and the model simultaneously.

It seems reasonable, then, to believe that, by varying the devices employed to make plane geometry intelligible, that is, by changing the style of presentation rather than the content of the course, the great mass of ordinary pupils would be able to cope with much of the material commonly reserved only for the select few. With traditional figures like the following (Fig. 1 or Fig. 2), for example, it is not at

[2] Bernardo, J. V., "A Helpful Technique in Teaching Solid Geometry," The Mathematics Teacher, xxxiii (January 1940), p. 39.

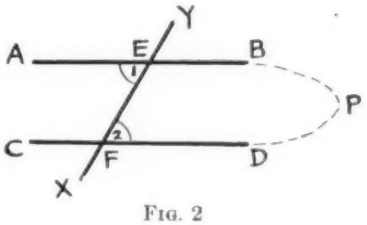

Fig. 2

all surprising that many pupils saw the proof of the equal alternate-interior angles theorem as through a glass darkly, if at all. This theorem in the conventional texts usually followed the exterior angle of a triangle theorem accompanied by a figure like Fig. 3. Obviously, it is not the learner's fault that the connection between Fig.

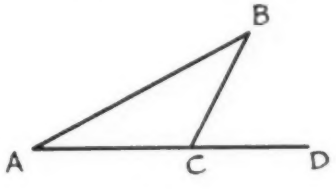

Fig. 3

3 and Fig. 1 or Fig. 2 does not stand out—no opportunity is afforded for the comprehension of the relationship between the two theorems. In Fig. 1 or in Fig. 2, *PEF* no more suggests a triangle than does an ultra "abstract art" painting resemble the common objects of everyday life.

Following the external angle of a triangle theorem with a figure like Fig. 4, on the other hand, provides a "means" to the "end" (the proof that if the alternate-in-

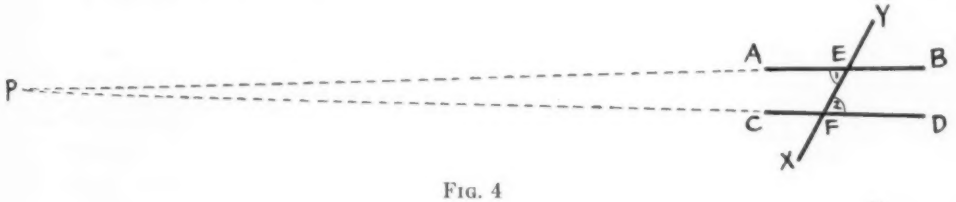

Fig. 4

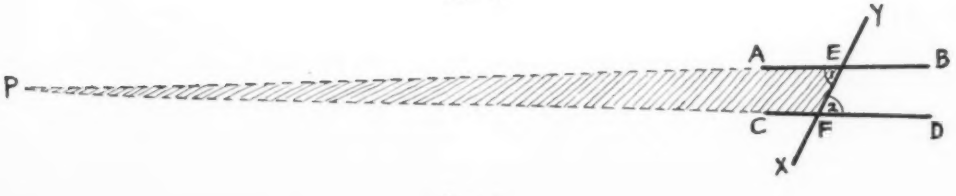

Fig. 5

terior angles are equal, the lines are parallel). Here, an insight into the fact that *PEF* is a triangle, and that angle "2" is an exterior angle is given a chance. A little shading in the figure (Fig. 5), or better still, a little color in *PEF*, a color different from that of lines *AB*, *CD*, and *EF*, will increase the chances for the emergence of insight still more.

Similarly in the case of other theorems that pessimistic course revisionists recommend be postulated or labeled for omission —this view, which regards individual differences as a challenge to better teaching techniques, holds that, with the proper kind of presentation, they could be mastered by any high school boy or girl, average as well as superior. The pupil who does not respond intelligently to the proofs of the congruence theorems, for instance, might suddenly "see the light" if the form of the statement were changed or if the "given" parts were indicated by marks on the sides and in the angles of the figure instead of in written form in the hypothesis alone. Others might require preliminary superposition exercises with angles of various kinds having sides of varying lengths, then a graduated transition to triangles made of transparent plastic or of rigid wire, before they could understand the demonstration from a drawing of the figure. Sometimes such a simple change as a change in the naming of the triangles from "*ABC* and *A'B'C'* " to triangle *ABC* and triangle *OXY* is a great help. Constructions may be depicted by a series of figures—one drawing for each step in the process—instead of a single figure showing the completed job. Films, both slide and moving picture, relating to mathematics, represent still another approach to the problem of bringing plane geometry within the comprehension of the ordinary mind. No matter what the approach the final result would be the same for all.

Course revision of this type, in which the "means" not the "ends" would be various, puts a high premium upon ingenious pedagogy. It calls for experimentation, inventiveness, and resourcefulness. No longer is the contention that plane geometry is beyond the comprehension of the rank and file accepted as a justification for offering a superficial smattering of the subject. The major function of the teacher becomes the provision of situations that give meaning to subject-matter rather than eliminate or omit it. He or she must study the learner, and gauge the method of presentation in accordance with the learner's best mode of understanding—change the conditions of learning not seek out the easier parts of the traditional course. Such adjustments are somewhat difficult to devise and operate, but they hold out the possibility of initiating the ordinary student into the ways of thinking of one of the greatest mathematicians who ever lived.

How Do You Live?

You may bring to your office and put in a frame
A motto as fine as its paint,
But if you're a crook when you're playing the game,
That motto won't make you a saint.
You can stick up the placards all over the hall,
But here is the word I announce:
It is not the motto that hangs on the wall,
But the motto you live that counts.
If the motto says "Smile" and you carry a frown;
"Do it now," and you linger and wait;
If the motto says "Help" and you trample men down;
If the motto says "Love" and you hate—
You won't get away with the mottoes you stall,
For truth will come forth with a bounce—
It is not the motto that hangs on the wall,
But the motto you live that counts.

From "The Kalends" of *The Waverly Press*

◆ THE ART OF TEACHING ◆

Toward Better Graphs

By EDWIN EAGLE, *Napa, California*

THE USE OF graphical presentation is now wide-spread, and it is becoming increasingly popular and important as a means of presenting data and conveying factual information. Even a cursory comparison of today's newspapers, magazines, and scientific and commercial publications with those of a few years ago indicates a definite trend toward an increased use of graphic methods. Yet the construction and use of statistical graphs is at present inadequately treated in our mathematics textbooks. Taking account of the recognized dependence of teachers, and especially of mathematics teachers, on textbooks it might be assumed that the topic is being inadequately presented in our classrooms. Such seems to be the logical conclusion of a careful analysis of twenty-nine mathematics textbooks of the seventh, eighth, and ninth grades, which, rightly or wrongly, are the only grades that devote any considerable time to this topic. These twenty-nine textbooks, all published since 1934, are those which are most widely used in the schools of California and also in other states. This weak spot in our instruction in mathematics should certainly be a matter of serious concern for educators.

Though a comparatively new technique, the statistical graph has had a period of very rapid development. According to H. Gray Funkhouser, of Columbia University, it is only about a century and a half old, having been invented in 1786 by William Playfair, a draftsman of James Watt, the inventory of the steam engine. By 1914 its effectiveness was recognized to the extent that we find a prominent engineer, W. C. Brinton making the following statement:

> Though accurate data and real facts are valuable, when it comes to getting results, the manner of presentation is ordinarily more important than the facts themselves; . . . If the facts were put in graphic form, not only would there be a great saving in the time of the readers, but there would be infinite gain to society, because more facts would be absorbed, and with less danger of misinterpretation.

In 1939 we find the Progressive Education Association recognizing the importance of graphic methods in their publication, *Mathematics in General Education* as follows:

> The graphical method of presenting data has recently become so prevalent that the ability to interpret the common types of graphs occurring in newspapers, magazines, and books may be considered as a necessary extension of the basic reading skills.

Though we have realized the importance of the topic, we are forced to admit that the presentation of it in our textbooks is not at a very high standard. This inadequacy exists, not because information is lacking, and not because techniques have not been developed, but because a wide discrepancy exists between the textbook presentation of the topic and the best standards that have been established and accepted. Since 1914, numerous books on graphic methods have been published which contain many important and fundamental principles, which should be but have not been incorporated into our textbooks. Several important organizations, national, and some even international in scope, have considered the matter of graphic procedures at numerous conferences. Most recent of these has been a series of meetings sponsored by the American Society of Mechanical Engineers. These have resulted in several important

publications, the last one being *Time-Series Charts, A Manual of Design and Construction*, in 1938, which has been accepted by the American Standards Association.

An analysis of the practices and standards accepted by such organizations reveals many procedures which are of real merit, not only because following them would make for greater uniformity, but because the procedures suggested make for effective graphs which give vivid, logical and correct impressions of the facts pictured. On the other hand, examination of the twenty-nine mathematics textbooks previously mentioned reveals that in regard to many graphical procedures we in the schools are not presenting to our youth the latest and best information, nor are we having them practice and use the procedures which will best develop in them the ability to use the graph as an effective tool. The material pertaining to graphs, as presented in these textbooks, in some respects does not correspond to the standards referred to above; many items of importance are entirely disregarded, many more are seriously slighted, and in a few instances actual contradictions exist. We can definitely improve our instruction in this field by taking into account the following items which are emphasized in the publications cited:

1. *The graph should have a title which should be complete and clearly stated, so that misinterpretation would be impossible.*

Keeping in mind that the fundamental purpose in constructing a graph is to portray facts and convey information, the importance of this item is obvious. Obvious too, to anyone who has had experience with students of junior high school age, is the fact that unless they are instructed concerning such an item and the reason and importance of it they will fail to take account of it in their work. Students do not just find out such things for themselves. Yet we find that two of the textbooks failed to mention the title and twelve made only casual reference to it without instructing the student to be sure that he puts one on his graphs. By being insistent in regard to this point we can materially raise the standard of student work.

2. *The source of the numerical data used should be indicated on the graph. In many cases it is wise to show the data also.*

What has been stated under item 1 can be repeated here. Further, living as we are in an age when we are assailed continuously with propaganda, bad and good, it is highly important that our students learn to check the sources of the information which they use. For this reason, as well as for the sake of completeness, it is quite essential that they be taught to indicate the source of the data used when constructing graphs, and that they become intelligently critical in regard to this point of graphs which they study and use in their school work or which they notice in newspapers, magazines, and books. Startling it seems that not one of the twenty-nine textbooks mentioned this point or even indicated by illustrations that it was an essential part of any graph.

3. *In picturing areas or volumes to represent quantities, care should be exercised to prevent giving false impressions. This applies especially to pictograph area and volume pictures.*

The danger here is quite obvious, but the error nevertheless occurs frequently. How often we see in newspapers, magazines, and books pictographs contrasting two different sized figures, such as a large soldier beside a small one. In such cases, comparing the heights of the two figures would give, let us say for illustration a ten to one ratio. Then an area comparison would give a one hundred to one ratio, and a volume interpretation would give a thousand to one relationship. Students should be taught to understand the ambiguity of such graphs (or grafts) and should be taught to be critical in interpreting them, as some of this type are deliberately designed to give a misleading impression. Only five of the twenty-nine textbooks called attention to the principle

here involved, while two of them contained as samples of correct practice illustrations which violate it.

All of this does not imply that pictographs should not be used. On the contrary, correct pictograph methods should be taught and encouraged. The fallacy of the commonly used type referred to above can be easily avoided by making the figures all the same size and basing the comparison on the relative number of figures shown in rows or columns. When so arranged, much of the advantage of the multiple bar graph results, but the graphs have greater eye appeal and the added advantage of the helpful suggestion of the pictograph figures. H. M. Bell's recent book, "Youth Tell Their Story," makes extensive use of such graphs as does Dr. Otto Neurath's "Modern Man in the Making." Construction of the pictograph figures can be greatly facilitated by cutting stencils from stencil paper or from heavy drawing paper.

4. *On bar graphs and line graphs the zero line should be included or its omission clearly indicated.*

Failure to observe this will result in a graph that gives a false impression. To illustrate, suppose the numbers 14 and 12 are to be represented. If we start with our base line at ten, then 14 will be represented four units above the base or twice as high up as the 12, thus suggesting an entirely false relationship. True, by reference to our marginal scale we can determine the correct relationship, but our graph nevertheless conveys a wrong impression; the point of making a graph in the first place has been lost. If, in some particular case, it is the differences and not the absolute amounts that are important, then the base line need not be the zero line, but this should be clearly and conspicuously indicated. This not only eliminates the false impression mentioned above but also emphasizes the fact that the differences and not the absolute amounts are what is important.

In regard to this principle as applied to the bar graphs the textbooks all illustrated correct practice, though only two of them discuss the false impression resulting from violating it. When it comes to line graphs, twelve of the textbooks included graphs drawn incorrectly with the zero line omitted, and one of these went on to explain that this is a correct and effective thing to do.

5. *The scales on a line graph should be so arranged and so numbered that an increase is represented by a rising of the line and a decrease by a dropping of the line.*

We have become accustomed to speaking of "the price going up," "the rising temperature," etc., and to associate such change with the directional concept. For conveying an impression that will at once suggest the desired relationship it is necessary to apply this principle in making line graphs. Though the graphs in the textbooks examined were drawn with this in mind, none of them included explanations of the manner of arranging and labeling the axes to achieve this result. The authors apparently assumed that the students would "catch on" by looking at a few graphs drawn in the proper manner.

6. *On multiple bar graphs the direction of the bars should be consistent with the data represented.*

Here again most of the textbooks, 16 of the 29, illustrated correct practice but did little else to teach it to the student. Eleven of the texts disregarded this item while two included illustrations which violated it.

The relative merits of horizontal bars and vertical bars has long been argued. Advocates of the vertical bars point out the advantages of the rise and fall in the tops of the bars indicating increases and decreases, this point being in harmony with the principle of the line graph. The greater ease of labeling the horizontal bars, especially for graphs to be printed for publication is a point in favor of that arrangement. There is some evidence to indicate that we judge horizontal bars more accurately, possibly because we have been more concerned with horizontal distances.

The lateral relative position of our eyes may have an effect on this also. Neither arrangement has any great advantage over the other so we are free to base our selection in any particular case on the basic principle behind graphical representation, that of presenting as logical a picture as possible. If a bar graph is to represent quantities or objects that are measured vertically upward, such as the height of mountains or oil derricks, vertical bars of course suggest the actual facts being pictured. Depth of wells, or distances below sea level would more logically be portrayed by bars extending vertically downward. Quantities measured horizontally, such as distances required to stop a car, distances broad jumped, length of rivers, etc., would be pictured by horizontal bars, ordinarily extending, for convenience, from left to right, as we are accustomed to read that way. In rare cases, as in graphing distances measured westwards, such as distances of islands in the Pacific from the California coast, the bars would extend from right to left.

7. *A type of graph should be selected that logically portrays the information.*

All too often students are left with the impression that the type of graph to be used is a matter of purely arbitrary selection. The contrary is usually the case. Parts of a whole should ordinarily be represented by one whole bar divided into parts, or by a pie chart. For comparison of several quantities, the multiple bar graph or the broken line graph is appropriate. For continuous change the curved line graph best conveys the desired impression. Besides teaching pupils these principles we must give them practice in making such selection instead of instructing them to picture a particular set of data by means of a certain type of graph. Only two of the textbooks definitely guide the student into exercises where he must use his judgment in selecting the type of graph to be used.

8. *The use of appropriate colors and shadings should be taught and encouraged.*

Since careful choice in colors and shadings can definitely aid in suggesting and conveying to the reader the desired picture or impression attention should be called to the usefulness of this device. Through our daily contacts we come to associate certain colors with various things; e. g., red with danger, or with being "in the red," black with "painting a dark picture of things," green with the favorable or with freshmen, etc. In graphs pertaining to data or objects that have definite color aspects, such as coal, grain, apples, copper, etc., the use of proper suggestion through the use of color should not be disregarded. Twenty-five of the textbooks studied totally disregarded this, the other four making only casual reference to it.

9. *Artistic contrast and artistic composition should be emphasized.*

The relative width of the grid lines, the axes, and the curves themselves; the width and spacing of the bars on bar graphs; the size and location of the title; the labeling and the numbering of the axes; the selection and the arrangement of colors and shadings; all can have an important effect on the appearance, the attractiveness, and the usefulness of the graph. All of this of course implies much attention to detail, much painstaking work by the student, and careful supervision of the work by the teacher, but it is nevertheless worth while. The student will profit far more from carefully and correctly completing one graph than from carelessly scratching off of a dozen of poor calibre. While all but two of the textbooks mention artistic contrast and artistic composition of graphs, only two of them give any helpful suggestions or instructions to make the students more effective in this regard.

In addition to teaching students principles of graphic representation which make for effectiveness and which are consistent with approved practice and established standards, we need to do more to make the learning functional. In this connection teachers of mathematics might well remember that their pupils are studying other subjects in which many situations

develop where quantitative information and relationships are important. Such situations provide excellent opportunities for the students to utilize graphic methods to more clearly analyze the situations for themselves and to more effectively portray them to their classmates. Also, we can help matters much by centering the study more around problems of direct interest to the students. Instead of having them draw graphs of "the hourly temperatures on a certain day in a large city," we might have them use graphs to study relationships between outdoor temperatures and temperatures in various parts of the school building. Or they might keep graphic records of various aspects of interscholastic athletic programs, or of the standings of the different salesmen in the school annual sale contest, or the number of students participating in each school activity compared to the relative expense of each. If we can have them use graphs to picture information that is significant and important to them they will come to think of the graph, not as an end in itself nor as an assignment to be done, but as a means of expression and as an effective language which they may use. As they become proficient in this mathematical a spect of language they will become more capable of thinking through the quantitative aspects of the many problems they will meet, and they will be able to more effectively convey their ideas to others; all of which will make them more competent in their work of adult life and as citizens of the community.

Professor E. R. Breslich Honored

WHEREAS: Ernest Rudolph Breslich, Associate Professor Emeritus of the Teaching of Mathematics, the University of Chicago, has shown a profound and abiding interest in the Central Association for more than three decades, presenting reports and papers on our Mathematics Section programs, 1915, 1916, 1925; publishing articles in our official journal *School Science and Mathematics*, 1920, 1934, 1935, 1936; Vice-President and President of our Association, 1926–1927; and

WHEREAS: he is esteemed as one of America's great teachers of mathematics, his kindly attitude, evenness of temper, and personal interest in students winning for him the title: "the teacher without an enemy"; joint-creator, with E. H. Moore and G. W. Meyers, of the Correlated Mathematics Movement in this country; a textbook writer of national influence; a Past Vice-President and Director of the National Council of Teachers of Mathematics; Be it therefore

RESOLVED: that the Officers and Board of Directors of the Central Association of Science and Mathematics Teachers delight to honor Professor E. R. Breslich by electing him an Honorary Member of this Association.

In the Year of our Lord MDCCCCXXXXI,
November 21.

Resolution prepared and presented by Edwin W. Schreiber, Professor of Mathematics, Western Illinois State Teachers College, Macomb, Illinois.

EDITORIALS

Mathematics in an Air Conditioned Civilization

In this complex civilization school administrators and teachers alike are faced with the problem of providing a better type of education for the seven million or so pupils who now make up our secondary school population. This means, not only drastic changes in the education of those pupils of the academic type to meet changing demands, but also an altogether different type of curriculum for those pupils who, under the traditional system "leave school without skills or orientation of any kind with respect to social institutions."[1]

We have all heard the familiar remark "We live in a changing world." The fact is that, at the present time we not only live in a changing world, but the rate of change is so much more rapid than previously that few people are willing even to predict with any degree of precision just what will come to pass in the next fifteen or twenty years.

What changes have taken place since the book "Around the World in Eighty Days" was written! People used to joke about the title. Recently, however, Secretary Knox, of the Navy Department in Washington, flew to Honolulu and back at a rate never dreamed of a generation ago.

This generation and the one to follow are going to be "air-conditioned," so to speak, and the sooner everybody adjusts his life to this situation the better for everybody concerned. The fact is that we must become air-conditioned in order to survive.

The significance of all this for science in general and mathematics, in particular, is obvious to all who think seriously and fairly about the situation. These subjects, or at least their fundamental and useful aspects, should be "must" studies for most of our school population in the days to come.

Those who have been thinking about the matter already realize that in any well organized course in air navigation many of the principles of mathematics, physics, and the other sciences previously taught in the secondary school can be applied in ways that will make these subjects interesting, meaningful, and more useful than ever before. This does not mean that the schools should immediately go over to a program of war mathematics only to drop it after the war is over. It means a careful reorganization of content material for teaching purposes which, after the emergency is over, will be just as useful in the days of peace that lie ahead. For after all, the science and mathematics that go into the construction of an army bomber is much the same as that necessary to build a commercial plane. We need to teach pupils the fundamental parts of the various divisions of mathematics that will equip them to make sensible applications in life in the most efficient manner.

If we are to furnish an adequate basis for the education of pupils in the fundamental principles of aeronautics, then we shall have to get rid of the "deadwood" (the debris) that clutters up the courses in mathematics, physics, and other subjects in our secondary schools. Thus, in order to have time for some of the real life applications of mathematics to air navigation and other similarly important life interests, we shall have to eliminate all such obsolete processes as factoring, complicated processes with polynomials, and the like from the secondary school. Moreover, this can be done without any loss to any important

[1] See Judd, Charles H. "The Real Youth Problem," *School and Society*, January 10, 1942, p. 32.

interest. But progress along this line is too slow. We need action and we need it quickly.

Here are some illustrations of the possible applications of mathematics that may be made to aeronautics at the present time on the secondary level.

The fundamental processes with integers and fractions both common and decimal which can be taught in the first six grades of the elementary school will furnish a basis for the real applications which should follow in the secondary school. Arithmetic itself has changed greatly. It is now dynamic where formerly it was static. It now deals with automobile tires, airplanes, and so on. The subject correlates well with algebra, informal geometry, and numerical trigonometry all of which are very important in solving the problems of air navigation.

The main use of algebra is made in connection with the formula or the rule behind it. Here aviation furnishes many interesting illustrations. For example, for any given speed, there is a certain altitude above ground at which a plane can fly in a circle around a pylon (or point on the ground) at the proper degree of bank with the lateral axis pointing directly toward the pylon at all times. This particular altitude is called the *pivotal altitude*[2] for that particular speed. The pivotal altitude is the same no matter what the radius of turn. The greater the radius the less the necessary bank, but at the pivotal altitude the lateral axis can always point directly toward the pylon.

The formula for the pivotal altitude is $P.A. = 0.067A^2$ in which P.A. is the pivotal altitude in feet and A is the speed of the plane in miles per hour. Thus the pivotal altitude varies as the square of the speed. If you double the speed the pivotal altitude is quadrupled. If $A = 100$ miles per hour, $P.A. = 670$ feet, and if $A = 200$ miles per hour, $P.A. = 2680$ feet.

[2] See Pope, Francis and Otis, Arthur S. *Elements of Aeronautics*, World Book Company, p. 23.

Obsolete problems about pipes running in and running out of cisterns, those about motion of unimportant objects, and other similarly outmoded problems should be relegated to the algebra museum and their places should be taken by problems about when, under certain specified conditions, two airplanes will meet, or how long it will taken an airplane to perform a certain specified function and then return to the carrier from which it took off in the first place. Such problems are found in the book referred to above.

We live in a world of three dimensions and for the most part, teach a geometry of flatland. There is no reason why the most important parts of solid geometry should not be combined with those of plane geometry and all be taught in the secondary school. What we need is a course in general mathematics running all through the secondary school.

The law of The Parallelogram of Forces and the theorems relating to similar triangles are both very important and useful in air navigation and should be taught in the geometry course in such a manner that useful applications can be made by the pupils. Scale drawings (an application of the idea of similar triangles) is commonly used in all navigation problems. Very little is done with such things in the mathematics classes in most schools.

Great use is also made of graphic charts of all kinds, both statistical and mathematical graphs, in air navigation. A great deal of this work is elementary and yet it is most important. It should receive more attention in secondary schools. Moreover, the work should be begun in the seventh grade.

Beginning in the ninth grade or earlier we need to teach all pupils a short unit of numerical trigonometry. It is one of the best liked topics in all of the year's work once the pupils have a chance to get it. Later this can be extended to include the more advanced stages of the subject including the fundamental ideas of spherical trigonometry for the abler pupils. For

most of the problems of aviation included in a recent book[3] a straightforward application of sines, cosines, and tangents is sufficient. For example, suppose we take the simple but meaningless problem sometimes found in textbooks:

> If the angle of elevation of a cloud bank as seen from a point A on level ground is 47°, what is the height of the cloud directly above a point B?

[3] Ibid.

Finding the height of such a cloud is now not only a real but an important problem in aviation and it can be solved by the use of a simple trigonometric formula.

These few illustrations will give a fairly complete idea of the range of mathematical applications in the aviation field. It is hoped that all teachers of secondary mathematics will begin to include such simple problems in their courses.

W.D.R.

Ulysses Grant Mitchell

Dr. Ulysses Grant Mitchell, professor of mathematics at the University of Kansas since 1905 died on January 1, 1942. Professor Mitchell served as principal of the high school and superintendent of schools at Hillsboro, Kansas, from 1898 to 1902, and as superintendent of schools at Lyons, Kansas, from 1902 to 1904. He went to the University of Kansas in 1905 as instructor in history. He served as instructor in mathematics from 1906 to 1908. From 1908–10 he held a teaching fellowship at Princeton, whence he returned in 1910 to the University of Kansas as assistant professor of mathematics. In 1915 he was made associate professor and in 1920 full professor and head of the mathematics department. Dr. Mitchell was 69 years of age when he died.

Professor Mitchell was one of those rare professors of mathematics who took a keen interest in mathematical education on the secondary level. He served on the recent Joint Commission of The Mathematical Association of America and The National Council of Teachers of Mathematics on "The Place of Mathematics in Secondary Education." He was a man of many interests, of a kindly nature, and was loved by all who knew him. His passing is a great loss to the field of mathematics.

W.D.R.

The Torch Bearer

By Nellie P. Bradley

Hold high the torch; You did not light its glow;
'Twas given you from other hands, you know.
'Tis only yours to keep it burning bright,
Yours to pass on when you no more need light.
For there are little feet that you must guide,
And little forms go marching by your side;
Their eyes are watching every tear and smile,
And efforts that you think are not worth while
May sometimes be the very helps they need,
Actions to which their souls would give most heed,
So that in turn they'll lift it high and say,
'I watched my mother carry it this way."

If brighter paths should beckon you to choose,
Would your small gain compare with all you'd lose?
Then lift the torch; You did not light its glow;
'Twas given you from other hands you know.
I think it started down its pathway bright
The day the Maker said, "Let there be light."

IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

The American Mathematical Monthly

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2. Fawcett, Harold, "A Mathematics Program with an Emphasis on General Education," pp. 25–31.
3. Keller, M. W., and Shreve, D. R., "Abilities of University Students in Freshman Mathematics," pp. 38–46.
4. Spitzer, Herbert F., "A Device as an Aid in Teaching the Idea of Tens," pp. 65–68.
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NEWS NOTES

With army officials warning educators to speed up training and to offer more short terminal courses to help the all-out war effort of the nation, the Commission on Junior College Terminal Education has decided to provide from coast to coast this summer three workshops for junior college instructors interested in setting up terminal courses and in studying other problems of terminal education. These workshops will be located on the east coast at Harvard University, in the midwest at the University of Chicago, and on the west coast at the University of California.

At least 100 scholarships will be provided for qualified junior college faculty members interested in study at the workshops.

Junior college heads representing the nation's 650 two-year colleges shifted their annual meeting from Los Angeles February 28 to Baltimore New Year's weekend in order to map out an immediate course of action for their institutions in wartime. They agreed that their first duty was to help win the war and pledged prompt cooperation to the government and all educational groups in order that "concerted action may prove valuable to the cause of victory and an intelligent humane after-the-war period."

On December 10, 1941, the Mathematics Section of Westchester County, New York, held its annual dinner meeting at the Birch Brook Inn, Bronxville, New York. In all fifty-six mathematics teachers and their guests attended.

Following an excellent dinner, the assembled group listened to Dr. William S. Schlauch of the School of Commerce, New York University, who spoke on the timely subject "The Mathematics of Some Business and Social Problems of Today." In his usual inimitable manner, Dr. Schlauch held his audience spellbound for the allotted time and was called upon to answer numerous questions in the way of an encore.

EARL R. KESSLER, *Chairman*

At the November meeting of the Men's Mathematics Club of Chicago Professor Nicolas Rashevsky, Associate Professor of Mathematical Biophysics, University of Chicago, will speak on the general topic, "Mathematical Biophysics." Dr. Rashevsky is highly trained in mathematics, biology, and physics, and has made many appearances before groups interested in mathematics. The talk was illustrated by lantern slides.

S. F. BIBLE, *Secretary*

The Department of Economics of Columbia University has adopted the following paragraph for insertion in the announcement of the Faculty of Political Science. The department hopes that it will be brought to the attention of students interested in economics at a sufficiently early stage to lead them to acquire at least the mathematical preparation indicated while they are still undergraduates. It should be noted that this is not a requirement, but is advice offered to prospective graduate students in economics; and also that some economic studies make use of mathematics substantially more advance than calculus and higher algebra.

"*Mathematical Preparation:* The use of mathematics, including higher mathematics, has become important in several branches of economics and statistics. Much of the recent important literature of general economics iswritten in a language not easily understood without some knowledge of the differential and integral calculus. Students planning to work for the Ph.D. degree in Economics will therefore find it advantageous to acquire familiarity with the calculus and with higher algebra before beginning their graduate studies in Economics."

HAROLD HOTELLING

SERVICES OF A LIVE MATHEMATICS CLUB

The Euclidian Club of East High School, Denver, Colorado shows what a live Mathematics Club may contribute in the way of service to the school, community and nation.

According to an article appearing in the *East High Spotlight*, the school paper—"The Euclidian Club, which is an organization devoted to Mathematics, is selling Defense Stamps at East, as a service to the school and to the United States Government."

"H. W. Charlesworth, sponsor of the club reports a total of $665.20 worth of stamps sold during the first week, or an average of 30 cents for every student at East." Each week a group of four club members take over the responsibility of the stamp sales. The club will continue to sell the stamps throughout the school year.

The Fourth Annual Mathematics Exhibit which will be held this year April 8 to 11 has not only become a tradition at East; but has attracted attention both locally and nationally.

This year the boys gave a mathematical face as an assembly program. The girls of the club plan a special program in the near future to boost the sale of Defense Stamps.

It is the policy of the club to give special recognition to outstanding mathematics students.

This year, especially the club members are alert to present mathematical needs, both local and national. That national defense and long time planning in times of peace put a great demand on mathematics is recognized by the Euclidians and their sponsor, all of whom are rallying to the cause with a practical and enthusiastic response.

The Christmas meeting of Phi Chi Mu (Decatur High School mathematics club, Decatur, Alabama) was in the form of a social, a "Mathematical Banquet." This Christmas function was "mathematical" in the decorations, menu, program, and every appointment. The decorations on the table included, as the main center ornament a Christmas tree trimmed with geometric figures, red candles in silvered candle holders, silvered pine cones, and, booklets containing the menu and program, all of which were made by the boys of the club.

The menu with every item suggestive of mathematics was as follows:

Semi-spheres of Grape-fruit
Cylinderical Celery
Elliptical Olives
Fraction of Turkey
Miscellaneous Dressing
Spherical Peas and *Carats*
Rice *Grains*
Cranberry Jelly *Stars*
Cardinal Aspic
Squares of Saltines
Plane Rolls with *Solid* Butter
Circle Coffee
Sector of *Pi*
with *Triangular* Cheese
Acute Indigestion *Q. E. D.*

Place cards, made by a clubster, were in the form of a T-square crossed with the triangle having the Greek letters Phi, Chi, and Mu in the three vertices respectively. (For several meetings the boys have had instruction in the fundamentals of mechanical drawing).

The program was as follows:

Toastmaster ..... James Holesapple
Invocation ..... Brad Bailey
Greetings ..... James Holesapple
Music ..... Billy Hulsey, John Cook, Eugene Wright
*Proof of a Theorem ..... Fletcher Eddens
Star Spangled Banner ..... In Unison

* Note: The theorem proved was a joke pulled in a spirit of fun. The boy who proved it selected a couple (one of the members and the girl who was his guest) and proved that the girl loved the boy, giving it as he would a formal proof of a geometry theorem.

Phi Chi Mu officers are:

President ..... James Holesapple
Vice-President ..... Edward Purser
Secy.-Treas. ..... L. W. Johnson
Program Chairman ..... Fletcher Eddens
Sponsor ..... Elizabeth Eich

The nation's first full-time program of instruction and research in applied mechanics began its Fall session at Brown University on October 1, 1941 where for eight months a group of experts will study mathematical problems in defense industries, especially aviation.

A small but carefully selected group of 25 students has been accepted to work with the staff of six American and European authorities in the field of higher mathematics, Dean Roland G. D. Richardson of Brown's Graduate School said. Seven courses are being offered, as compared with four courses given during the experimental session this summer.

These courses will deal with specialized problems in mathematics which bear upon aviation and aeroplane construction, with shipbuilding, gun manufacturing, armor-plating on tanks and armored cars, optical instruments, and other weapons and instruments of war requiring a high degree of technical knowledge to plan, design and make.

The students enrolled include a seismologist intending to enter aeronautics, a weather bureau research man with technical aspirations, a life insurance statistician who wants further training for defense industry, college and university professors, and graduate students seeking dedefense jobs in aviation and other fields.

"Few laymen realize how vital is the application of higher mathematics to many problems in industry," Dean Richardson said today. "A full exploitation of aerodynamics and other fields bearing directly upon defense activities awaits the basic work of mathematicians.

"Bedfore the war the principal work in applied mechanics was carried out in Europe, especially in Germany. Now that we cannot have the benefit of German studies, we must do the work ourselves and make up for lost time. Obviously there is a tremendous amount of ground to be covered."

The courses being offered at Brown are "Numerical and Graphic Methods of Applied Mechanics," "Partial Differential Equations," "Elasticity," "Advanced Dynamics," "Fluid Dynamics," "Conference on the Theory of Air-

flight," and "Special Conferences in Advanced Topics and Research."

In addition to lectures, considerable time will be given over to individual and group conferences. Some students are expected to bring with them particular problems which require mathematical solutions.

Dean Richardson announced that the following have been appointed to the teaching staff this year:

John L. Synge, professor of applied mathematics at the University of Toronto, who is engaged in research for the National Research Council of Canada. Prof. Synge is an authority on the applications of tensor calculus to geometry and mathematical physics.

Richard von Mises, head of the Department of Aeronautics in the Graduate School of Engineering at Harvard, and formerly director of the Institute for Applied Mathematics at the University of Berlin. He is an expert in applied mathematics and the theory of aeronautics.

Jacob D. Tamarkin, professor of mathematics at Brown and editor of the international journal, Mathematical Reviews. He is an expert in differential equations.

Willy Feller, assistant professor of mathematics at Brown and formerly of the Universities of Kiel, Copenhagen and Stockholm. He is an expert on graphical and numerical methods in mathematics, and on foundations of probability and statistics.

Stefan Bergman, expert in applied mechanics, who was formerly an instructor at the Institute for Applied Mathematics at the University of Berlin and at the Technological Institute of Tomsk.

Willi Prager, professor of engineering at the University of Istanbul and formerly associated with the Aerodynamical Institute of Gottingen. He is an expert in the theory of structures. Prof. Prager has not yet arrived in this country.

The Cooperative Committee on Science Teaching

Under the sponsorship of five scientific societies representing Biology, Chemistry, Mathematics, Physics, and Research in Science Teaching a committee has been formed to work on educational problems of vital interest to all science teachers which no single organization can solve working alone. The committee is known as the Cooperative Committee on Science Teaching.

Two meetings have been held, one in April and one in November, 1941. Work is now in progress on four problems:

1. Licensing or certification of secondary-school science teachers. The committee hopes to work out a solution that will be practicable, and that will be adopted by certification authorities.

2. The college training of prospective science teachers. It is desirable to prepare teachers for teaching certain combinations of subjects rather than to prepare intensively in one subject.

3. Exploratory studies of the secondary-school science curriculum. The Committee hopes to stimulate a number of colleges and universities to organize workshops and conferences for bringing together secondary-school teachers to work on their educational problems.

4. Problems of state or local agencies needing the services of educational consultants on questions pertaining to science teaching. The committee offers its services as a consultant to state or local agencies working on problems pertaining to science teaching.

The Committee consists of the following members:

Representing the American Association of Physics Teachers
K. Lark-Horovitz, Purdue University
Glen W. Warner, Wilson Junior College, Chicago

Representing the American Chemical Society
B. S. Hopkins, University of Illinois
Martin V. McGill, Lorain High School, Lorain, Ohio

Representing the Mathematical Association of America
A. A. Bennett, Brown University
Raleigh Schorling, University of Michigan

Representing the National Association for Research in Science Teaching
G. P. Cahoon, Ohio State University
Robert J. Havighurst, University of Chicago

Representing the Union of Biological Societies
Oscar Riddle, Carnegie Station for Experimental Evolution

Robert J. Havighurst is Chairman of the Committee and Glen W. Warner is Secretary.

Mathematics for Prospective Members of Aircraft Crews

An Appeal To Teachers

There is an urgent need for the expansion of the R.A.F. young men who are physically and temperametally fit for the service are unfortunately lacking in the necessary knowledge of elementary mathematics. For some time the Regional Committees for Adult Education have been giving assistance to such men, but the time has now come when this hlp must be greatly increased and extended to men of lower initial attainments. Cases differ so much that only very small classes are possible, or individual tuition may be necessary. Usually the

only time that the men are free is in the evening.

A syllabus of the minimum course is appended:

Fractions—problems;
Conversions for Metric to British units;
Averages, Ration and Proportion, Percentages;
Relation between the sides of a right angle triangle and square roots (proof not required);
Transposition of equations;
Problems using simple equations;
Graphs (not algebraic);
Triangle of velocities.

Will teachers who are willing to help in this vital national service please communicate with a Regional Committee (address c/o the nearest University or University College) or the local Director of Education?

Curriculum Bulletins of Interest to Teachers and Supervisors and Curriculum Builders of Science and Mathematics

The University of Oregon Curriculum Laboratory has just announced several new pamphlets in its *Curriculum Bulletin* series. *Curriculum Trends and Recommendations for a 12-Year Mathematics Program* (No. 39, 30¢) includes outlines of several recent outstanding courses and a model course. *A Proposed Junior High School Curriculum* (No. 13, Revised, 15¢) is another pamphlet which presents a typical model program.

Insurance: A Unit for Social Mathematics (No. 33, 20¢), is rich in suggested activities references, etc., and includes social and economic aspects.

Two other pamphlets valuable for enriching an educational program are: *A Framework for American Educational Philosophy* (No. 42, 25¢), which serves as a guide and outline to stimulate thinking in the development of an educational point of view; and *The School Curriculum and Community Life* (No. C, 25¢), which includes a basic philosophy, principles for developing a community-centered curriculum, and an outline of a typical program.

These pamphlets may be obtained through the University Cooperative Store, Eugene, Oregon. A complete annotated list of bulletins now available may be secured from the same source. The subscription price to the CURRICULUM BULLETIN is $5.00 per year.

Teachers, College, Columbia University will offer the following courses in the teaching of mathematics this summer, July 7 through August 14, 1942:

By Professor A. Day Bradley: Navigation, professionalized subject matter for teachers. Professor John R. Clark: Teaching algebra in secondary schools: Secondary school mathematics for non-academic pupils. Mr. Gordon R. Mirick: Elementary mechanics with special reference to dynamics. Professor W. D. Reeve: Teaching and supervision of mathematics: junior high school; Teaching and supervision of mathematics: senior high school. Professor Carl N. Shuster: Modern business arithmetic; Mathematics applied to elementary military engineering. Dr. R. R. Smith: Teaching geometry in secondary schools; Teaching informal geometry in junior high schools. Miss Ethel Sutherland: Teaching arithmetic in primary grades; Teaching arithmetic in intermediate grades.

There will be held during the Summer Session, on consecutive Thursdays beginning July 9, five informal conferences in which all the instructors in mathematics will take part for the purpose of bringing before the students vital questions affecting the present reorganization of mathematical instruction. There will be opportunity for discussion in which the students are invited to participate fully. Registration for these conferences is not necessary. The conferences and discussions will deal with current questions in the teaching of mathematics with emphasis on problems connected with the present war emergency.

The following bibliography on Navigation has been prepared with the help of Dr. Hale Pickett, and Dr. Carl Shuster both of whom are actually giving instruction in Navigation in their respective schools.—Editor

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18. Weems, P. V. H.: *Air Navigation*, McGraw-Hill Book Company, 1938, Price $5.00.
19. Coastwise Navigation (I) and Deep Sea Navigation (II). Published by Weems System of Navigation, 122 West 3rd Street, Los Angeles, California.
20. Digest of Civil Air Regulations for Pilots. C. A. Bulletin #22 (Third Edition) U. S. Government Printing Office, Price $0.20.
21. Ground Instructors Manual. Civil Aeronautics Bulletin #30, U. S. Government Printing Office, September 1940, Price $0.15.

The following letter, reproduced in full has recently been sent out by the Bureau of Navigation in Washington, D. C. With it was "Enclosure A" which is also reproduced here:

DEAR SIR:

The Navy Department has received many requests from high schools and preparatory schools throughout the country requesting information as to how they can best prepare their young boys for service in the Navy in this war. One of the greatest obstacles which the Navy has encountered in its program to secure and train men for service in the Navy has been the fact that high school and college applicants have not had the necessary instruction in certain essentials to permit them to be successfully trained in more technical fields. Lack of knowledge in the above essentials by trainees has hampered the Navy training program tremendously.

The Department therefore feels that the greatest contribution the secondary schools can make in defense of our country is to introduce certain courses of instruction in addition to those already available in the schools, and to make certain that the young high school or preparatory school boy is given this most vital preparation. The majority of secondary school graduates enter the colleges and universities of this country with only one year and a half preparation in mathematics and the sciences and great numbers do not even have this fundamental background. The demand of the Navy for properly equipped young men and boys is great, and this demand can only be met by proper scholastic preparation in the schools of the country.

Enclosed is a suggested outline of subject matter, and it is firmly and urgently hoped that you can see fit to assist the Navy by including all of some of the suggested subjects in your curricula.

Your whole-hearted cooperation in this matter will be greatly appreciated, and it is further requested that you disseminate the contents of this letter and enclosure to all high schools and preparatory schools, both public and private, inside your state.

Sincerely yours,
RANDALL JACOBS,
Rear Admiral, U. S. Navy
Chief of the Bureau of Navigation.

ENCLOSURE (A)

Suggestions for Instruction in High Schools and Preparatory Schools

1. Morse Code.
2. Elements of Telephone Communication.
3. Elements of Radio Communication.
4. Elementary Principles of Radio.
5. Plane Trigonometry.
6. Solid Geometry.
7. Quadratics.
8. Physics.
9. Chemistry.
10. Shop Mathematics.
11. Vocational Training.
    (a) Machine Shop Work.
    (b) Foundry Work.
    (c) Internal Combustion Engines (Gas, Diesel, and Aircraft).
12. Elementary Navigation.

At least two and one-half years of mathematics to include algebra, plane and solid geometry, and trigonometry should be given to male students.

A New Book of Unusual Interest

The TEACHING of SECONDARY MATHEMATICS

By CHARLES H. BUTLER
Department of Mathematics
Western Michigan College of Education

and F. LYNWOOD WREN
Professor of Mathematics
George Peabody College for Teachers

McGraw-Hill Series in Education. **514 pages, 6x9, $3.00**

In the perspective of a historical and philosophical setting the authors of this outstanding new book analyze the implications of current criticisms and recommendations pertaining to the mathematics curriculum and consider in detail many general and specific instructional problems.

The material has been organized in three main divisions:

PART I. THE PLACE AND FUNCTION OF MATHEMATICS IN SECONDARY EDUCATION. The discussion in this section is designed to be of special interest to the student of education in his desire to arrive at a proper orientation of mathematics in the secondary school curriculum.

PART II. THE IMPROVEMENT AND EVALUATION OF INSTRUCTION IN SECONDARY SCHOOL MATHEMATICS. Here the authors deal with those problems which concern the administrator and supervisor in their efforts to improve instruction in secondary mathematics.

PART III. THE TEACHING OF THE SPECIAL SUBJECT MATTER OF SECONDARY MATHEMATICS. This section discusses problems of instruction in arithmetic, algebra, geometry, trigonometry and calculus.

Write for further information

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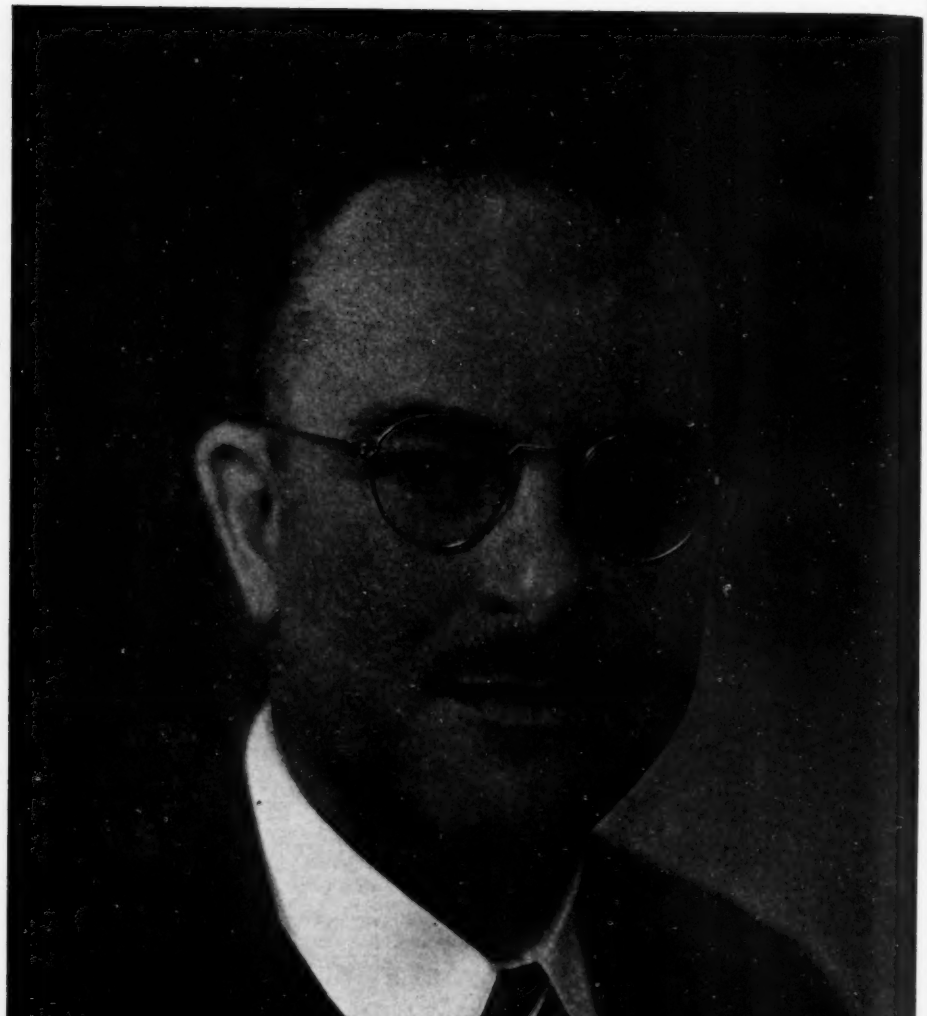

The New President of the National Council of Teachers of Mathematics

ROLLAND R. SMITH